How Sleeping Patents Can be Used?

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Abstract

Based on the fact of large sleeping patent portfolios, I develop a multi-staged static model to investigate the motivations of holding sleeping patents and its impact on innovation through the channel of mergers and acquisitions (M&As). With imperfect patent enforcement, the patent litigation outcome can be affected by sleeping patent holders through strategic M&As either with the plaintiff or the defendant, thus generating additional benefits. I show these benefits from potential patent litigation is a driving force for firms to hold sleeping patents and engage in M&As. Moreover, holding sleeping patents has a positive impact on innovation. This paper also gives policy implications in terms of the patent acquisition by patent trolls, patent holdup, and antitrust regulations.

Keywords: *sleeping patents, strategic patenting, research and development (R&D), patent litigation, mergers and acquisitions (M&As)*

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1 Introduction

Patent analysis is a longstanding subject in finance and economics as patents serve as the significant measurement for innovation output (e.g., Hall and Harhoff, 2012; Nagaoka et al., 2010; Abraham and Moitra, 2001; Encaoua et al., 2006). A patent is a governmental grant to inventors of a right to earn monopoly profits by excluding other competitors. However, there are two counter-intuitive facts observed in recent years. One is the mystery of holding large unused patents. Holding large patent portfolios are common in the high-tech industry. For example, a consortium of Apple, Microsoft, and other large corporations paid $4.5 billion for Nextel's patent portfolio of approximately 6,000 patents. Microsoft paid approximately $1 billion for nearly 1,000 patents from AOL in 2012, and then sold some of the patents to Facebook for $550 million. However, most patents are not brought to market, which are referred to as "sleeping patents". According to Palomeras et al. (2003), a large percentage of patents are unused in firms. For example, IBM, Philips and Siemens only use about 40% of their intellectual property portfolio in production. Torrisi et al. (2016) also find empirically a significant portion of patents are not used internally or for market transactions. Why would these companies pay a high price for large portfolios of patents, whilst the majority of which have no obvious current or future applications? What is the impact of patent system on innovation based on the fact of large holdings of sleeping patents?

The Second is the high litigation rate with the existence of patent thicket, which describes a dense web of overlapping patent rights, due to the substantial increase in patent fillings and strengthened links between different technologies (Shapiro, 2000). In this case, it is not optimal to file patent litigation due to the fragmentation of patent ownership, resulting in the possibility of being counter-sued. However, contrary to the expectation of low litigation rates, the number of patent litigation cases increases by 13% since 2019 in the US as reported by World Intellectual Property Review and most of cases are involved with sleeping patents. Do firms use patent litigation to gain additional synergies instead of preventing infringement with the presence of patent thicket?

Previous research argues the motive of holding sleeping patents arises from entry deterrence, the increase in subsequent innovation, or increased profits after acquiring large number of patents (e.g., Caskurlu, 2019; Phillips and Zhdanov, 2013). This paper aims to investigate the motivation of holding sleeping patents and the ex-ante impact on innovation and provide a theoretical explanation of the two counter-intuitive facts of large sleeping patent holdings and the rise in patent litigation with patent thicket. In this paper, I first show patent litigation can be a strategic decision through the channel of mergers and acquisitions (M&As) and examine whether and how are synergies created in patent litigation using sleeping patents. In addition, I further study the impact of holding sleeping patent on pre-merger innovation.

This paper is motivated by the real examples that firms engage in M&A to get litigation benefits from the acquired patents. One example is a legal dispute between Google and Oracle. Oracle sued Google for copyright

and patent infringement in August 2010. Oracle claimed that Google was aware that they had developed Android without licensing the Java programming language's application programming interfaces (APIs) owned by Sun before Oracle's acquisition in January 2010. While two District Court jury trials were found in Google's favour, the Federal Circuit Court overturned both decisions, claiming that APIs are copyrightable and Google's use of them violated the fair use defence. Google successfully petitioned the Supreme Court to hear the case during the 2019 term, with the emphasis on the copyrightability of APIs and subsequent fair use. The decision is made in April 2021 that Google's usage was indeed fair use [1].

Another famous example is the smartphone patents wars[2]. In 2012, Google announced an agreement to acquire Motorola Mobility. This acquisition potentially provides Google with leverage and ownership rights to protect the entire Android ecosystem, which alternative contractual structures cannot provide. Before the merger, other high-tech companies sued Google for infringement on its Android technology. In 2010, Apple, for example, sued Google-Android partner HTC for patent infringement on the iPhone. However, because Google has almost no patents on smartphone technology, its ability to solve this problem is currently limited. Motorola sued Apple for patent infringement in 2010, and Apple responded by counter-suing. Google acquired Motorola Mobility for $12.5 billion in 2012, in part to obtain Motorola's portfolio of thousands of patents, which it used to defend its Android mobile operating system against intellectual property claims made by Apple, Microsoft Corp., and others. These patents shifted the power balance in the smartphone war. In 2014, Apple and Google Inc's Motorola Mobility unit reached a settlement agreement[3].

These examples highlight a number of important aspects of the merger transactions that we investigate. First, merger participants pursue related R&D activities before the acquisition. Second, there may be a third party who gets involved in the patent disputes between two firms through M&A and is able to resolve the patent dispute. Third, there may be extra synergies gained through patent litigation after M&A. To understand whether these examples represent a general pattern underlying M&As, I investigate the following research questions in this chapter. How are firms' patent litigation strategies related to M&A incidence? How do patent litigation outcomes affect the bargaining position and target choice in M&A negotiations? Does the presence of post-merger patent litigation affect pre-merger innovation incentives for all patent holders involved? The central hypothesis is that patent litigation synergies are a significant driver of M&As.

To examine the role of patent litigation in M&As, I propose a static model where two firms competing in a product market have a patent dispute. Both firms arguably infringe the patents owned by a third party. Firms can choose to settle or start litigation to resolve their patent dispute separately, but if the third party holds sleep-

[1] "Oracle v. Google: What the verdict means for open source". InfoWorld. 2021-09-21. https://www.infoworld.com/article/3633668/oracle-v-google-what-the-verdict-means-for-open-source.html

[2] See the detailed lawsuits at https://en.wikipedia.org/wiki/Smartphone_patent_wars#2012.

[3] Daisuke Wakabayashi And Rolfe Winkler (16 May 2014) "Apple, Google End Patent Battle". The Wall Street Journal. https://www.wsj.com/articles/apple-google-end-patent-fight-1400283981.

ing patents instead of commercialised patents, he can get involved in the patent dispute between the two firms through M&A, thus reducing the total number of resolutions. The third-party can choose to merge with either the patent holder or the alleged infringer. Firms make decisions by maximising firm value in different cases.

This paper first shows patent litigation based on sleeping patents drives the incidence of M&A activities. A high win rate of sleeping patents increases the likelihood of M&A. This is because firms can earn a higher value in patent litigation after M&A. Second, M&A target choices depend on the comparative advantages in patent litigation gained through M&As. Specifically, the M&A outcome is a bargaining game between excluding the infringer for the patent holder and reducing litigation risks for the non-patent holder. As a result, the firm with strong sleeping patents tends to merge with the producing firm with strong patents to exclude the infringer and reduce the litigation risk between them. Alternatively, a strong sleeping patent holder merges with a non-patent producing firm to gain a better bargaining position by invalidating the other weak patents. The above two results provides the theoretical explanation for the first paradox of why firms hold sleeping patents.

Third, I find firms choose strategic patenting when the win rates for other patents in the market are high to avoid litigation risks from other patent holders. However, they start litigation after M&A to gain benefit. The likelihood of patent litigation rises as a result of patent holders' strong incentives to gain monopoly profits through lawsuits in the face of intense market competition. Therefore, firms choose strategic patenting when the win rates for other patents in the market are high to avoid litigation risks from other patent holders, but gain additional value in potential patent litigation after M&A. This result reconciles the second paradox of high litigation rates with patent thicket.

Fourth, through M&A, strategic patenting increases firms innovation incentives both for the sleeping patent holders and other commercialised patent holders. this result shows that patent system still promotes innovation even if no commercialisation exists in product market. Even if the commercialised patent holder is the outsider of M&A, he saves litigation costs due to the higher likelihood of cross-licensing after merger.

This paper contributes to the patent litigation and M&A literature, in at least three ways. First, this paper provides a novel explanation of why firms may often hold large sleeping patent portfolios, i.e., to gain additional litigation with the fact of patent thicket. I show patent litigation can be a strategic decision and a driver of mergers and acquisitions (M&As) by examining the difference between patent litigation outcomes and firms' abilities to change these outcomes after M&A. This is also the first study to examine whether and how are synergies created in patent litigation using sleeping patents. Second, This is the first study to discuss the impact of strategic patenting on R&D through patent litigation and shows the positive impact of strategic patenting on firms' R&D incentives. Third, I add to prior findings by showing the firm that has no special assets (i.e., the alleged infringer) can be the M&A target in some cases.

This paper proceeds as follows. Section 2 presents the related literature. Section 3 presents the baseline model

analysis in terms of M&A, commercialisation and R&D decisions. Section 5 extends the model to examine the firms' litigation decisions with multiple infringers. Section 6 presents comparative statics of the model, among other findings. Section 7 discusses the model assumptions and potential implications. Section 8 concludes.

2 Related literature

This paper is related to several strands of literature in law and economics, as well as innovation. The first is the motives of M&A activities. Hoberg and Phillips (2010) first show product market synergy is an important driver of M&As. Bena and Li (2014) show that the technological overlap is a source of synergies and that this drives the decision which firms to acquire and the post-transaction performance. Another reason for participating in M&A transactions, particularly in the most recent M&A wave and in industries such as information and communication technology (ICT) and pharmaceuticals, is technology and innovation (Chakrabarti et al., 1994; Sleuwaegen et al., 2006). There is a growing body of literature on how mergers affect innovation incentives, i.e., the interplay between mergers and acquisitions and innovation. Granstrand and Sjölander (1990) find large established companies buy small innovative start-up companies to source new technologies. Zhao (2009) shows that M&A is used for sourcing technologies externally. In contrast, Cunningham et al. (2021) argue that firms engage in M&As with innovative targets to discontinue the target's innovation projects for preemptive purpose. Moreover, Grimpe and Hussinger (2014) demonstrates that patents not only interpret a company's knowledge base but also have a preemptive power due to the legal title that allows the owner to prevent third parties from using the technology. They conclude that an intermediate level of technological relatedness results in the highest acquisition prices, and the target's preemptive value increases with the target's similarity to the acquirer's technological profile. Besides, Creighton and Sher (2009) find firms tend to resolve patent disputes through a merger. Few papers examine the impact of patent litigation on M&A incidences. Marco and Rausser (2008) empirically show that firms in Plant Biotechnology with overlapping technologies engage in M&A to avoid mutually blocking technology. Caskurlu (2019) empirically find that the losing defendants have a stronger incentives to merger with firms with substitute products. The previous papers focus on the impact of patent litigation on firms' profits from productions. To examine the overall impact of patent litigation on firms' M&A incentives, this paper adds to the literature by demonstrating that, except acquiring technologies or innovation from outside sources or obtaining a higher profits, firms have an incentive to engage in M&A to gain additional litigation benefits other than solving patent disputes.

In addition, studies that investigate the selection of M&A targets find that technologically close targets are preferred (Bena and Li, 2014; Grimpe and Hussinger, 2014). However, post-M&A evidence suggests that if the acquirer's goal is to improve innovation performance, they should choose firms that are neither too similar nor too distant (Cloodt et al., 2006). According to the resource-based view (RBV) and knowledge-based view (KBV)

(Kogut and Zander, 1992; Nelson and Winter, 1997) literature, M&A is a method of acquiring new competencies and knowledge. As a result, it predicts that in order to attract the interest of potential acquirers, target firms must possess valuable resources (e.g., patents, key technological personnel, superior IP management, and learning capabilities). Furthermore, empirical evidence suggests that firms with more financial constraints are more likely to be acquired, while financially strong firms are less likely to engage in a M&A transaction (Danzon et al., 2007). This paper adds to the literature by demonstrating that firms' M&A target selection is also influenced by patent rights and potential patent litigation outcomes.

This paper is also linked to literature on sleeping patents, patent portfolios and patent acquisition. The existing literature on strategic patenting has shown that patents may remain unused for strategic reasons, such as the entry deterrence of new competitors or the protection of business (Choi and Gerlach, 2017; Gilbert and Newbery, 1982; Scott Morton and Shapiro, 2016; Ziedonis, 2004; Palomeras et al., 2003; Torrisi et al., 2016). Furthermore, the innovation-hindering effect of strategic patenting has been studied in previous literature (e.g., Shapiro, 2003; Jaffe and Lerner, 2011; Blind et al., 2009; Gurgula, 2020). In addition, several studies find that firms accumulate patents to reduce litigation risk or to gain a comparative advantage in the market (e.g., Morton, 2012; Parchomovsky and Wagner, 2005; Dorsey, 2013). The recent studies has established the relationship between M&A and sleeping patents. Norbäck et al. (2020) theoretically find that if the patent protection is strong, large incumbent firms buy small firms who are not in the market to ensure that their patents are not used in the market. The closest work with this paper is Choi and Gerlach (2017), which develops a theoretical framework to discuss the impact of the strength of patent portfolio on cross-licensing, patent acquisitions, and innovation. However, Choi and Gerlach (2017) does not discuss the interaction of patent litigation and M&A, especially the case of merging with the non-patent holder to facilitate the cross-licensing and the patent acquisition between producing and non-producing firms. Building on previous research, I find that firms have incentives to engage in strategic patenting in order to gain litigation benefits through mergers and acquisitions, and that strategic patenting encourages firms to innovate.

3 The basic model

There are three firms with equal bargaining power in this multi-stage game. Firms A and C sell homogeneous goods in the product market[4] and each firm earns a duopoly profit π_a^d and π_c^d for firm A and C respectively, while the monopoly profit, which is higher than the sum of the duopoly profit, is denoted by π^m. The total profit in the duopoly market is denoted by $\pi^d = \pi_a^d + \pi_c^d$ and is assumed to be strictly smaller than the monopoly profit. Firm A has a patent while firm C does not. In addition, firm B has related but sleeping patents. As a result, two patent-holding firms A and B have patent disputes with each other, and they both have patent infringement

[4]To simplify the model, I assume the profits for firms are fixed.

conflicts with the firm C that has no related patents. Patents give the patent holders rights to exclude others from the market through patent litigation. However, patent litigation is expensive, time-consuming and complicated. Also, the legal rights to prevent others from producing are highly uncertain (Lemley and Shapiro, 2005). Therefore, settlement is the most common way to resolve patent disputes. In recent years, mergers and acquisitions (M&As) have become more common as a new method of resolving patent disputes (Creighton and Sher, 2009).

I assume M&A is not permitted between the two producing firms A and C under a very strict antitrust law. This is due to the fact that potential anti-competitive outcomes are more easily anticipated and detected in a horizontal merger between competitors. However, firm B can choose whether to engage in M&A with one of the two producing firms to resolve the patent dispute based on firm B's patent without raising antitrust concerns. The default case with no M&A is referred to as Case 1. If M&A occurs, all firms bargain to determine the M&A outcome through a three-party bargaining game. If two patent-holding firms A and B engages in M&A, this is Case 2. If the non-producing patent holder merges with non-patent holder, this is Case 3. The sequence of events is presented in Figure 1.

[Insert Figure 1 here]

To demonstrate the significance of patent litigation on firms' M&A incentives, I assume away any additional synergies associated with producing profits generated in M&As, resulting in a small difference between M&A and patent acquisition. In practice, other synergies associated with producing profits in M&A increase the value of merged firm and make M&As more appealing to firms than patent acquisitions.

Furthermore, different M&A outcomes influence the patent litigation outcomes, as well as firm values after mergers and value through Nash bargainings when negotiating M&As. In Case 2, the merged entity's win rate in patent litigation against firm C increases due to the merger of two patent portfolios owned by the two patent-holding firms. With the higher win rate, the merged firm has strong incentives to gain the monopoly profit by excluding the infringer C, thus increasing the likelihood of litigation. In Case 3, both parties, i.e., the merged firm BC and firm A, have the ability to threaten each other with patent litigation, so firms choose to settle to save litigation costs, reducing the risk of litigation. As a result, firms determine the optimal M&A outcomes in a three-party bargaining game by considering the maximum value of outside options, i.e. the firms values of resolving the patent disputes in other M&A cases.

Using backward induction, I first examine firms' optimal litigation or settlement strategies for resolving patent disputes in various potential M&A cases and the case without M&A. I use the notations V^{sue}, V^{settle} and V^{nl} to denote the firms value if the optimal strategy is litigation, settlement or doing nothing. In any case, the patent-holding firm first determines whether it is worthwhile to litigate by comparing the patent holder's value of litigation with the value if no action is taken. I assume the winning probabilities for the two patent holders in court

ruling are common knowledge and they are denoted by P_a and P_b for firm A and B respectively. Litigation is expensive and the cost for one lawsuit is denoted by L. If litigation is worthwhile and poses a credible threat to the defendant, both the plaintiff and the defendant can reach an agreement to avoid costly litigation and share market profits through settling ex-ante[5]. If a settlement occurs, the defendant pays the plaintiff the royalty payment, which is denoted by $r_i, i \in \{1, 2, 3\}$ in different M&A cases. In addition, each party pays the settlement cost, which is denoted by c, and is lower than the litigation cost (i.e., $L > c$).

To simplify the setting, I assume that all firms have sufficient revenues that are not related to the patents to afford the costs in terms of M&As, litigation and settlement which is denoted by v_i, where $i = A, B, C$. With this assumption, the challenger will not be forced out with the threat of litigation due to the inability to pay litigation costs, thus the challenger has a strong incentives to infringe[6]. Since there are no other synergies in M&A, if two patent holders merge, the profit that is not related to the patents are denoted by $v_{ab} = v_a + v_b$ and the profits from the patents are π_a^d for the merged firm AB. If firm B merge with firm C, the other revenues are denoted by $v_{bc} = v_b + v_c$ and the profits based on the patents are still π_c^d for the merged firm BC.

3.1 Patent litigation outcomes with and without M&As

In this section, I first analyse firms' patent litigation strategies in different M&A cases assuming patent-holding firm B does not commercialise his patents, i.e., firm B has related but sleeping patents. Since the main focus of interest in this paper is the incidence of M&A activities, I first start the analysis of firms' strategies in patent litigation if M&A occurs.

3.1.1 Case 2: M&A between A and B

In this case, the two patent-holding firms A and B can engage in M&A to resolve the dispute between each other and avoid litigation. The merged entity first decides whether to start expensive litigation against firm C. If litigation is worthwhile, the merged firm AB and firm C can choose to resolve the patent dispute through settlement. Figure 2 shows the different outcomes in Case 2.

[Insert Figure 2 here]

Through M&A, two firms' patent portfolios also merge together[7] and this increases the total win rate when litigating against the challenger C. Parchomovsky and Wagner (2005) argues as long as a portfolio of litigated patents is not completely the same with other patents in different portfolio, which, under the patent law, is impossible,

[5] Ex-post settlement is not considered in the model because in this static setting with the lump sum litigation cost, ex-ante settlement is a dominant strategy.

[6] Firms' financial constraints play a significant role in determining patent litigation and innovation strategies, so this is left for future research.

[7] This can also be achieved by patent acquisitions.

the win rate in patent litigation increases as the number of litigated patents increases, even if the average win rate based on any single patent is rather low. Intuitively, as a result, merging increases the merged firm's chance to exclude the non-patent holder C, and the merged firm AB's incentives to litigate against firm C are higher when the win rate for any of the single patent holders is low (i.e., low P_a or low P_b). Furthermore, when compared to settling with a single patent holder, challenger C has a stronger incentive to settle with the merged firm due to the credible litigation threat.

The merged firm AB can sue firm C for patent infringement with the winning probability P_m in court. If the merged firm wins the case, the infringer C is forced out of the market and the merged firm can earn monopoly profit π^m. Otherwise, with probability $1 - P_m$, the merged firm loses the case and two firms remain in the status quo as a duopoly[8]. As in Choi and Gerlach (2017), I assume the ex-post joint win rate of the combined firm after merging is $P_m = (1 - P_a)P_b + P_a$[9].

The merged firm would litigate if $V_m^{sue}(P_m) \geq V_m^{nl}$, which yields a litigation condition when the merged firm's expected joint revenue from judgement is higher than the litigation cost. When litigation is credible threat, firms can settle to resolve the dispute in order to save costs. Settlement is feasible, when the cost saving from settlement instead of litigation is higher enough for both the merged firm AB and firm C.

Overall, high win rate for patent-holding firms increases the merged firm value, thus increasing the M&A incentive between the two patent holders. Given any value of the rest of the model parameters, there always exist P_a and P_b, such that merged firm AB can obtain a higher firm value through patent litigation or settlement because the higher joint win rate P_m compared to the single win rate. When the joint win rate is high, the merged firm can gain high firm value via patent litigation. When the joint win rate is moderate, the merged firm can obtain additional royalty payment from the non-patent holder through settlement. For low joint win rate, firms have no additional value.

[8]With the assumption that the challenger has enough other revenues to afford the litigation costs, i.e., $V_c^{sue} > 0$, the challenger C's other revenues have a lower bound, i.e., $v_c > L - (1 - P_m)\pi_c^d$.

[9]According Choi and Gerlach (2017), the increase in the win rate for firm A due to the merging of two patents is $(1 - P_a)P_b$. The effect of acquiring the additional patent portfolio on the strength of the existing patent portfolio is decreasing in the original strength. For instance, if $P_a = 1$, there would be no impact on the strength of the patent portfolio. The results are robust to other complementary assumptions when merging two patent portfolios.

With the proof in Appendix A.1.1, the firms' value in Case 2 can be summarised as follows

$$V_m^2 = \begin{cases} v_{ab} + \pi_a^d + P_m(\pi^m - \pi_a^d) - L, & \text{if litigation for} \quad P_m > \frac{2(L-c)}{\pi^m - \pi^d} \\ v_{ab} + \pi_a^d + r_2 - c, & \text{if settlement for} \quad P_m \in [\frac{L}{\pi^m - \pi_a^d}, \frac{2(L-c)}{\pi^m - \pi^d}] \\ v_{ab} + \pi_a^d, & \text{if nothing for} \quad P_m < \frac{L}{\pi^m - \pi_a^d} \end{cases} \tag{1}$$

$$V_c^2 = \begin{cases} v_c + (1 - P_m)\pi_c^d - L, & \text{if litigation for} \quad P_m > \frac{2(L-c)}{\pi^m - \pi^d} \\ \pi_c - r_2 - c, & \text{if settlement for} \quad P_m \in [\frac{L}{\pi^m - \pi_a^d}, \frac{2(L-c)}{\pi^m - \pi^d}] \\ v_c + \pi_c^d, & \text{if nothing for} \quad P_m < \frac{L}{\pi^m - \pi_a^d} \end{cases} \tag{2}$$

where $r_2 = P_m\pi_c^d + \frac{1}{2}P_m(\pi^m - \pi^d)$.

3.1.2 Case 3: M&A between B and C

In this case, the sleeping patent holder B merges with the non-patent holder C and their patent dispute is resolved through merger. The merged firm BC and the patent holder A choose how to resolve their patent disputes, i.e., to settle or to litigate.

Both the producing firm A and the merged firm BC have related patents, thus both of them can start patent litigation. As assumed in Choi and Gerlach (2017), there is no additional litigation cost to counter-litigate after one of the firms starts the lawsuit, making counter-litigation a dominant strategy for both firms[10]. This implies that when firms decide whether to initiate litigation they take into account the possibility of being counter-sued.[11]. If litigation is worthwhile for at least one of the patent-holding firms, they can decide whether to form an ex-ante cross-licensing agreement to avoid costly litigation. Cross-licensing can be regarded as the settlement with lower joint costs. The cost after entering cross-licensing for each firm k is lower than the simple settlement cost, i.e., $k < c$. Firms' strategies in Case 3 are shown in Figure 3.

[Insert Figure 3 here]

Overall, in comparison to Case 2, if Case 3 occurs, all firms' incentives to initiate litigation decrease due to the threat of counter-sue. As a result, the non-patent holder who is likely to be excluded due to infringement benefits from this merging strategy.

[10]The model will be more complicated if considering the additional cost of counter-claim, but I do not expect it to have a significant impact on my main results. I leave this for future work.

[11]I also allow the possibility that litigation initiated by firm A has already taken place and merger occurs afterwards. Counter-claim can still occur after merger.

With the proof in Appendix A.1.2, the firm's expected profit can be written as follows:

$$V_m^3 = \begin{cases} v_{bc} + \pi_c^d - P_a\pi_c^d - L \\ \quad +P_b[(1-P_a)\pi^m + (2P_a-1)\pi_c^d - P_a k], & \text{if litigation for} \quad (P_a, P_b) \in \mathcal{L} \setminus S \\ v_{bc} + \pi_c^d + r_3 - k, & \text{if settlement for} \quad (P_a, P_b) \in \mathcal{L} \cap S \\ v_{bc} + \pi_c^d, & \text{if nothing for} \quad (P_a, P_b) \notin \mathcal{L} \end{cases} \tag{3}$$

$$V_a^3 = \begin{cases} v_a + \pi_a^d - P_b\pi_a^d - L \\ \quad +P_a[(1-P_b)\pi^m + (2P_b-1)\pi_a^d - P_b k], & \text{if litigation for} \quad (P_a, P_b) \in \mathcal{L} \setminus S \\ v_a + \pi_a^d - r_3 - k, & \text{if settlement for} \quad (P_a, P_b) \in \mathcal{L} \cap S \\ v_a + \pi_a^d, & \text{if nothing for} \quad (P_a, P_b) \notin \mathcal{L} \end{cases} \tag{4}$$

where $r_3 = [P_a(1-P_b) + P_b(1-P_a)]\pi_a^d - P_a(1-P_b)\pi^m + \frac{1}{2}[P_a(1-P_b) + P_b(1-P_a)](\pi^m - \pi^d)$.

3.1.3 Case 1: No M&A

In this case, no M&A is allowed[12]. Patent disputes are likely to occur among these three firms and they choose to resolve their disputes through litigation or settlement[13]. With the proof in the appendix A.1.3, I show the optimal strategies in a two-stage sequential game. In the first stage, two producing firms A and C decide their optimal strategies to resolve their patent dispute, i.e. doing nothing, litigation, or settlement. In the second stage, depending on the outcome in stage 1, firm B chooses whether to settle with the one or two firms in the market with the credible litigation threat. If settlement occurs, I use $\hat{V}_i, i \in \{A, B, C\}$ to denote the firm i's value. Otherwise, firms' values are denoted by $V_i, i \in \{A, B, C\}$. Specifically, if firm A wins and gains the monopoly profit in the first stage, firm B settles with firm A only if B can impose a credible litigation threat. If firm A loses, firm B can settle with both firms A and C if litigation initiated by firm B is a credible threat. The game tree of this sequential litigation is presented in Figure 4.

[Insert Figure 4 here]

This figure demonstrates that firms' strategies are more complicated than those in the previous M&A cases. M&A has a two-fold effect on patent litigation. First, in Case 2, M&A can save the royalty payment between firm A and B while also increasing the win rate in litigation against C. Moreover, in case 3, M&A can provide firm B

[12]Here, non-producing firm B acts like a non-practising entity (NPE) that fails to produce or does not produce intentionally, and firm A or C are practising entities (PEs). This model also incorporates the possibility of firms whose business model is purely based on extracting revenues from licensing their patents with the threat of patent litigation, which is called "patent trolls".

[13]Here, I assume the patents owned by A and B are related but this does not mean firm A and B should be in the same industry. One kind is opportunistic patent litigation that is documented in Bessen and Meurer (2005), which refers to patent lawsuits which rely on weak patents to induce licensing. A patent suit is weak if the objective probability of successfully proving infringement and overcoming defences, such as patent invalidity, is low at the time of filing. In addition, Bessen and Meurer (2013) also find there is a substantial percentage of patent lawsuits that occur between distant firms. This suggests that there may simply be too many patent holders that pose a litigation threat but who have dissimilar technologies and products.

with the opportunity to gain monopoly profit while reducing firm C's litigation risk from A. Second, M&A exposes firm B to the potential litigation from firm A, which can be eliminated in Case 1.

Using backward induction, I first analyse the non-producing firm B's strategy. Since firm B is not producing, it is optimal to settle with other firms to earn royalty payment instead of litigation due to the lower costs in settlement (see proofs in appendix A.1.3). The royalty payment the firm B can get from firm $i \in \{A, C\}$ is denoted by r_b^i. According to the proofs in Appendix A.1.3, the settlement occurs when the royalty payment is higher than the litigation cost, i.e. $r_b^i \geq L$. Note, when firm B is able to settle with two producing firms, the total litigation costs are reduced when suing two firms that produce homogeneous goods. Therefore, the settlement condition is $r_b^a + r_b^c \geq L^0$ where $L^0 \in (L, 2L)$ is the reduced total litigation costs.

I assume the profit earned by the patent holder is higher than the infringer (i.e. $\pi_a^d \geq \pi_c^d$), so the royalty payment paid by firm A is higher than that by firm C, and firm B has a higher incentive to settle with firm A. By taking firm B's litigation incentives into account, the total royalty payment based on firm B's win rate P_b can be expressed as

$$r_b = \begin{cases} P_b \frac{\pi^d}{2}, & \text{if settle with A\&C for} \quad P_b \geq \frac{2L^0}{\pi^d} \\ P_b \frac{\pi_a^d}{2}. & \text{if settle with A for} \quad P_b \in [\frac{2L}{\pi_a^d}, \frac{2L^0}{\pi^d}) \\ P_b \frac{\pi^m}{2}. & \text{if settle with the monopolist A for} \quad P_b \in [\frac{2L}{\pi^m}, \frac{2L}{\pi_a^d}) \\ 0. & \text{if no settlement for} \quad P_b < \frac{2L}{\pi^m} \end{cases} \tag{5}$$

In the first case when $P_b \geq \frac{2L^0}{\pi^d}$, firm B can settle with both firms because it has a credible litigation threat against both A and C with litigation costs L^0. Second, when $P_b \in [\frac{2L}{\pi_a^d}, \frac{2L^0}{\pi^d})$, firm B is able to settle with firm A only because its litigation threat against A is credible whereas that against C is not. Third, when $P_b \in [\frac{2L}{\pi^m}, \frac{2L}{\pi_a^d})$, firm B can settle with firm A only if A gains the monopoly profit after patent litigation against C. Finally, if $P_b < \frac{2L}{\pi^m}$, the win rate is too low to threaten with patent litigation, thus settlement with either firms is infeasible. Overall, the non-producing patent holder firm B's win rate affects the royalty payment and also the other patent holder firm A's incentives to litigate at the first place.

Following the same steps, in the first stage, the patent holder firm A can choose whether to litigate or not litigate. If the litigation threat is credible (i.e., P_a is high enough), then both litigants can negotiate to reach the settlement agreement and the challenger pays the royalty payment to the patent holder A to avoid the costly litigation. I then discuss firm A's incentive to litigate by taking into account the royalty paid to firm B in the next stage as expressed in Eq(5).

With the proof in Appendix A.1.4, we can obtain firms' values in the four cases depending on royalty expressed in Eq(5).

$$V_i^3 = \begin{cases} V_i^{settle}, & \text{if settlement occurs} \\ V_i^{sue}, & \text{if litigation occurs} \\ V_i^{nl}, & \text{if nothing occurs} \end{cases} \tag{6}$$

where $i = a, b, c$ and V_i^{settle}, V_i^{sue} and V_i^{nl} are expressed in Appendix A.1.4.

3.2 M&A incentives if B is not a patent holder

To investigate how firm B's patent litigation affects firms' incentives to engage in M&A, I assume that firm B does not own any related patents and thus has no right to exclude other firms from the market, and then examine non-competitive firms' M&A incentives (i.e. between A and B or between B and C). Only firm A and C have patent dispute and they settle or litigate depending on P_a.

In this case, M&A with firm B does not change the win rate for firm A or C in court ruling. Obviously, firms A and B have no incentive to engage in M&A with B when there is no additional litigation benefits gained through M&A.

Producing firms have an incentive to engage in M&A activities with a sleeping patent holder, particularly when the combined party can utilize the sleeping patent to change the litigation outcome, e.g., increasing the royalty payment in settlement or the likelihood of winning in litigation. On the one hand, by merging with firm B, the patent holder A increases her chances of excluding the non-patent holder and saves costs associated with resolving the dispute with the sleeping patent holder. On the other hand, by merging with firm B, the non-patent holder C can reduce the risk of litigation initiated by firm A and thus gain benefits if the sleeping patent is strong. Therefore, in my model, the sleeping patent holder B determines his optimal M&A strategy by weighing the costs and benefits of merging with one of the producing firms and rejecting the other. The section that follows provides a detailed examination of firms' M&A strategies in a simplified three-player bargaining game.

3.3 M&A decisions

In this section, I discuss the equilibrium M&A given any set of parameters. That is, whether firm B chooses to merge (i.e. Case 1 or not) and if so, which firm should B choose to merge with (i.e. Case 2 or 3). Firm B makes his M&A decision by comparing his values in M&A determined by Nash bargaining in a simplified three-party bargaining game.

In a standard three-party bargaining game, only two of the three players can reach the agreement and all firms

have to choose which agreement they will form, which is similar to my setting since the three firms in the market cannot merge together. In this case, each player's outside option in Nash bargaining with one party is its value in agreements with another party, but none of the parties know their exact value of outside option before bargaining. In my model, firm B is the party who makes the decision on whom to merge and firm A and C cannot merge together, it simplified the standard three-party bargaining game. The standard three-party bargaining problem can be solved in either cooperative approach or non-cooperative approach (Binmore and Eguia, 2017; Roth et al., 1985). Here, I choose the cooperative approach based on Bennett et al. (1995). This is because first, there is a unique Nash solution in the cooperative model, whereas the non-cooperative bargaining models often have multiple equilibria. Second, the solutions in the proposal-making model, which is the non-cooperative approach in the three-player game context and a multi-stage game with time punishment[14], also converge to the unique solution in cooperative model, when the time punishment is high.

Based on Bennett et al. (1995), the simplified three-player bargaining game includes four stages. In the first stage, I solve the Nash solution for any two firms negotiating M&A separately while ignoring the existence of the third party. These Nash solutions do not take into account the possibility of the other M&A agreement, thus is called "constrained Nash bargaining solutions". They are denoted by N_i^2 in Case 2 or N_i^3 in Case 3 for firm $i \in \{A, B, C\}$. The outside option when calculating the constrained Nash bargaining solutions is Case 1, i.e., no M&A, detailed in Section 3.1.3 and is denoted by $N_i^1 = V_i^1$. In the second stage, I identify the "Nash stable agreement" and the "Nash unstable agreement" as in Bennett et al. (1995). The Nash stable agreement, where firms' anticipated bargaining values will not be affected by other agreements, is the one with the highest constrained Nash bargaining solution of the two-party bargaining for firm B solved in the first stage, i.e., $\max\{N_b^1, N_b^2, N_b^3\}$. Note that, if Case 1 is the Nash stable agreement, firms have no incentives to engage in M&A, so the final M&A outcome is Case 1. The Nash unstable agreement is the M&A case with lower Nash solution for firm B. In the third stage, firms may be willing to change how they split the merged firm value in the Nash unstable agreement if they can earn more in M&A than no M&A by taking into account their values in the other M&A case (i.e. the Nash stable agreement). For example, if Case 2 is the Nash stable agreement, firm B and C can negotiate their values in Case 3 by considering the outside option is the Nash stable agreement, instead of no M&A. The firms' bargaining values in updated Nash unstable agreements are denoted by $\bar{N}_i$. In the last stage, I determine the final M&A outcome, which is the agreement that firm B can earn a higher value by comparing his value in the Nash stable agreement and in the updated Nash unstable agreement. This agreement is called Nash dominant agreement as in Bennett et al. (1995) and the corresponding firms' values are denoted by ND_i. Overall, the non-producing firm B's value in M&A bargaining depends on the value of outside option and the value of the outside option is affected by the M&A firms' value in different litigation outcomes, which is discussed in the above sections.

[14] It takes time to make proposal and players are impatient, so the utility of future payoffs are discounted (see Epple and Riordan, 1987; Bennett et al., 1995).

In the first stage, I obtain a simple two-party Nash bargaining solutions when the outside option is Case 1. Each firm that in the M&A agreement earns a proportion of the merged firm's value and pays the M&A cost c^m. Therefore, the constrained Nash bargaining solutions for firms in Case 2 are

$$N_b^2 = (1-\omega_a)V_m^2 - c^m, \tag{7}$$

$$N_a^2 = \omega_a V_m^2 - c^m, \tag{8}$$

where ω_a is the proportion earned by firm A and firm C's value is V_c^2 as discussed in Section 3.1.1. The proportion ω_a can be solved through the maximisation problem, that is

$$\max_{(N_a^2, N_b^2)} [N_a^2 - V_a^1]^{\frac{1}{2}} [N_b^2 - V_b^1]^{\frac{1}{2}}.$$

The constrained Nash bargaining solutions for firms in Case 3 are

$$N_b^3 = (1-\omega_c)V_m^3 - c^m, \tag{9}$$

$$N_c^3 = \omega_c V_m^3 - c^m, \tag{10}$$

where ω_c is the proportion earned by firm C and firm A's value is V_a^3 as discussed in Section 3.1.2. ω_c is solved from the maximisation problem, that is

$$\max_{(N_c^3, N_b^3)} [N_c^3 - V_c^1]^{\frac{1}{2}} [N_b^3 - V_b^1]^{\frac{1}{2}}.$$

Substituting these proportions, I obtain the constrained Nash bargaining solutions as follows:

$$N_a^2 = \frac{1}{2}(V_m^2 - V_b^1 + V_a^1) - c^m, \tag{11}$$

$$N_b^2 = \frac{1}{2}(V_m^2 - V_a^1 + V_b^1) - c^m, \tag{12}$$

$$N_c^3 = \frac{1}{2}(V_m^3 - V_b^1 + V_c^1) - c^m, \tag{13}$$

$$N_b^3 = \frac{1}{2}(V_m^3 - V_c^1 + V_b^1) - c^m, \tag{14}$$

$$N_i^1 = V_i^1. \tag{15}$$

The first four expressions show firms' values in different M&A cases when they ignore the possibility of other M&A case (i.e., the outside option is no M&A). In general, firms' bargaining values constitute two components. The first term is the surplus a firm can obtain through merging compared to not merging and two firms equally split this surplus based on their equal bargaining power. The second term is the M&A cost. The last expression shows firms' values without M&A.

In the second stage, I identify the Nash stable agreement and the Nash unstable agreement by comparing N_b from Eq(11). There are three possibilities of the Nash stable agreement, that are Case 3, Case 2 or Case 1. First, if $N_b^3 \geq \max\{N_b^1, N_b^2\}$, the Nash stable agreement is Case 3. This condition also ensures that $N_c^3 \geq V_c^1$. The merger between B and C (i.e. Case 3) is stable because firm B prefers their payoff in Case 3, to payoff in Case 2. The Nash unstable agreement thus is Case 2. Second, if $N_b^2 \geq \max\{N_b^1, N_b^3\}$, the Nash stable agreement is Case 2 and Nash unstable agreement is Case 3. Third, if $N_b^1 > \max\{N_b^2, N_b^3\}$, the Nash stable agreement is Case 1. Since all firms earn more in Case 1, no firm has the incentives to engage in M&A and the final outcome is Case 1.

In the third stage, I investigate whether firms are willing to update their values in the Nash unstable agreement for the first two possibilities.

First, the Nash stable agreement is Case 3. If the condition $N_b^2 < N_b^1$ holds, which is equivalent to have $N_a^2 < N_a^1$, it indicates firm B has the incentive to merge in Case 2 because the value of merging in Case 2 is higher than the value without M&A. It may be worthwhile for firm B to break the bargaining in the Nash stable agreement with the non-patent holder C (i.e., give up N_b^3) and form the agreement with the patent holders A. Therefore, the outside options when determining the updated Nash bargaining solutions in Case 2 are N_b^3 and V_a^3 for firm B and A respectively.

Similarly, the updated Nash bargaining values for firm B and A in Case 2 are

$$\bar{N}_b^2 = (1 - \bar{\omega}_a)V_m^2 - c^m, \tag{16}$$

$$\bar{N}_a^2 = \bar{\omega}_a V_m^2 - c^m, \tag{17}$$

where $\bar{\omega}_a$ are the proportion of merged firm earned by firm A in the updated agreement. By solving the maximisation problem, i.e.,

$$\max_{(\bar{N}_a^2, \bar{N}_b^2)} [\bar{N}_a^2 - V_a^3]^{\frac{1}{2}} [\bar{N}_b^2 - N_b^3]^{\frac{1}{2}},$$

I obtain the updated Nash bargaining solutions as follows:

$$\bar{N}_a^2 = \frac{1}{2}(V_m^2 - N_b^3 + V_a^3) - c^m, \tag{18}$$

$$\bar{N}_b^2 = \frac{1}{2}(V_m^2 - V_a^3 + N_b^3) - c^m. \tag{19}$$

The first term in the above expressions shows that firm equally divide the surplus if merging in Case 2 not Case 3 and the last term is the M&A cost.

Second, the Nash stable agreement is Case 2. If $N_b^3 < N_b^1$ and $N_c^3 < N_c^1$, firm B and C may update their bargaining values in Case 3 by considering the values of the outside option are their values in Case 2 (i.e., N_b^2 and

V_c^2). Following the same methods as above, the updated Nash bargaining values for firm B and C in Case 3 are

$$\bar{N}_b^3 = (1 - \bar{\omega}_c) V_m^3 - c^m, \tag{20}$$

$$\bar{N}_c^3 = \bar{\omega}_c V_m^3 - c^m \tag{21}$$

where $\bar{\omega}_c$ are the proportion of merged firm earned by firm C in the updated agreement and can be solved through the maximisation problem

$$\max_{(\bar{N}_c^3, \bar{N}_b^3)} [\bar{N}_c^3 - V_c^2]^{\frac{1}{2}} [\bar{N}_b^3 - N_b^2]^{\frac{1}{2}}$$

Thus, the firms' values in the updated Nash unstable agreement (i.e. updated Case 3) can be expressed as

$$\bar{N}_c^3 = \frac{1}{2}(V_m^3 - N_b^2 + V_c^2) - c^m \tag{22}$$

$$\bar{N}_b^3 = \frac{1}{2}(V_m^3 - V_c^2 + N_b^2) - c^m. \tag{23}$$

In the last stage, I determine the M&A outcome by identifying the Nash dominate agreement. If firm B's value in the Nash stable agreement is higher than that in the updated Nash unstable agreement (i.e., $N_b \geq \bar{N}_b$), the Nash stable agreement is the Nash dominate agreement. Otherwise, if firm B's value in the Nash stable agreement is lower than that in the updated Nash unstable agreement (i.e., $N_b < \bar{N}_b$), the updated Nash unstable agreement is the Nash dominate agreement, which is the result of strategic M&A.

In the above sections, I examine firms' litigation strategies in various M&A cases, namely, mergers between two patent holders, mergers with the non-patent holder, and no M&A, and how they determine their M&A outcomes based on their firm values. The analysis reveals that the outcomes of patent litigation vary significantly across M&A transactions. If M&A occurs, on the one hand, by merging with the patent holder, the patented technology is highly protected and, depending on the degree of market competition $\pi^m - \pi^d$, the litigation risk is increased. As a result, the infringer is very likely to be forced out by future patent litigation. On the other hand, by merging with the non-patent holder, the merged firm and the patent holder are more likely to maintain the status quo through an ex-ante cross-licensing agreement. Otherwise, if no mergers and acquisitions occur, firm B's win rate increases the likelihood of litigation or settlement between firms A and B. In general, M&A reduces litigation costs but does not reduce litigation risk, especially if M&A occurs between two patent holders.

Knowing firms' M&A decisions and potential patent litigation strategies with sleeping patents, I further relax the assumption by allowing the possibility of commercialisation for firm B and analyse firm B's decision of whether to sleep or commercialise the patents in the following section.

4 Commercialisation

To investigate when firm B engages in strategic patenting, i.e. having sleeping patents, I modify the model to allow firm B to choose between commercialising the patents and holding sleeping patents. Firm B make his decision by comparing his value with sleeping patents (i.e. ND_b) that is discussed in Section 3.3 and the value with commercialised patent that will be discussed in this section. The firms' value if B commercialises are denoted by $V_i^0, i \in \{A, B, C\}$. Therefore, by allowing the possibility of commercialisation by firm B, firms, values can be expressed as:

$$V_b = \begin{cases} V_b^0, & \text{if} \quad V_b^0 \geq ND_b \\ ND_b. & \text{if} \quad V_b^0 < ND_b \end{cases} \tag{24}$$

$$V_a = \begin{cases} V_a^0, & \text{if} \quad V_b^0 \geq ND_b \\ ND_a. & \text{if} \quad V_b^0 < ND_b \end{cases} \tag{25}$$

$$V_c = \begin{cases} V_c^0, & \text{if} \quad V_b^0 \geq ND_b \\ ND_c. & \text{if} \quad V_b^0 < ND_b \end{cases} \tag{26}$$

If firm B commercialises the patents and enters the market to compete with the producing firms A and C, both firms A and B have patents related to their products. This fragmentation of patent ownership on the overlapping technologies captures "patent thicket" in the law and economics literature (Shapiro, 2000).

I assume firms' profits if firm B commercialises are denoted by π_i^0 for firm $i, i \in \{A, B, C\}$. I assume $\pi_i^0 < \pi_i^d$, which also implies the sum of the profits in the market denoted by π^0 is lower then the total duopoly profits π^d. The monopoly profit is still π^m. Due to the antitrust concern, none of these firms are allowed to merge with each other, so they have to resolve their patent disputes though litigation other than M&A. Similar to Case 3, two patent-holding firms first choose whether to litigate against each other by considering the possibility of counterclaim. To accommodate various scenarios, I assume firm A or B initiates patent litigation against the other two firms which are potential infringers with cost $L^0 \in (L, 2L)$ based on the fact the firms are able to pay litigation costs using other revenues[15]. If the litigation is a credible threat, firms choose whether to settle to save litigation costs. Figure 5 shows the sequence of events and the corresponding outcomes.

[Insert Figure 5 here]

[15]Since firms are selling the homogeneous good, if firm A wins the litigation against firm B, she can also win against firm C in a patent infringement lawsuit. Vice versa, if B wins in the patent lawsuit against A, he can also win firm C for patent infringement (Haley, 1993). If the patent-holding firm starts the litigation against two infringers, the worst case is to continue to share the market and the best case is that he can regain the monopoly profit. If the litigation cost is not too high to afford, it is optimal for firms to litigate both infringers. It is easy to verify this optimal strategy if the cost of litigation is lower than $\frac{\pi^0 - \pi_a^0}{\pi_c^0} L$.

4.1 Litigation

If a firm decides to initiate patent litigation against two infringers by considering the possibility of counter-sue, there are four different cases depending on the win rate P_a and P_b. First, with probability P_aP_b, both firms' patents are found to be valid in court, thus firm A and B can engage in a cross-licensing agreement with cost k for each firm and gain the duopoly market profit π_a^d and π_b^d respectively. Second, with probability $P_b(1-P_a)$, firm B wins the case but firm A loses, and firm B's profit increases to π^m, while firm A and C are out of the market. Third, with probability $P_a(1-P_b)$, firm A wins the case but firm B loses, so both firm B and the C stop producing the good, while firm A's profit increases to the monopoly profit π^m. Last, with probability $(1-P_b)(1-P_a)$, both firms' patents are proved to be valid in court and neither firms infringe, so three firms share the market profits.

Firm A, thus starts litigation against two infringers with litigation cost L^0 by taking into account the counter-litigation by the other patent-holding firm B if

$$V_a^{sue} = v_a + (1-P_a)(1-P_b)\pi_a^0 + P_aP_b(\pi_a^d - k) + P_a(1-P_b)\pi^m - L^0 \geq v_a + \pi_a^0.$$

Firm B litigates by taking into account the counter-litigation, that is

$$V_b^{sue} = v_b + (1-P_a)(1-P_b)\pi_b^0 + P_aP_b(\pi_b^d - k) + P_b(1-P_a)\pi^m - L^0 \geq v_b + \pi_b^0,$$

The litigation set for firm A and firm B can be defined as

$$\bar{\mathcal{L}}_a = \{(P_a, P_b) | P_a[(1-P_b)\pi^m + (P_b-1)\pi_a^0 + P_b(\pi_a^d - k)] - P_b\pi_a^0 - L^0 \geq 0\}$$

and

$$\bar{\mathcal{L}}_b = \{(P_a, P_b) | P_b[(1-P_a)\pi^m + (P_a-1)\pi_b^0 + P_a(\pi_b^d - k)] - P_a\pi_b^0 - L^0 \geq 0\}.$$

Litigation occurs when $(P_a, P_b) \in \bar{\mathcal{L}} = \bar{\mathcal{L}}_a \cup \bar{\mathcal{L}}_b$, because I assume a counter-sue without additional costs will always occur if the patent holder is sued.

Both firms have incentive to litigate if $V_a^{sue} + V_b^{sue} \geq v_a + \pi_a^0 + v_b + \pi_b^0$, i.e.,

$$\overbrace{(P_a(1-P_b) + P_b(1-P_a))(\pi^m - \pi_a^0 - \pi_b^0)}^{\text{expected revenue when only one patent holder wins}} + \underbrace{P_aP_b(\pi^d - \pi_a^0 - \pi_b^0 - k)}_{\text{expected revenue when both firms win}} \geq 2L^0. \tag{27}$$

Firm C's value of litigating is $V_c^{sue} = v_c + (1-P_a)(1-P_b)\pi_c^0 - 2L$.

By comparing the inequality (67), which is the condition that both patent holders have incentives to litigate

when the sleeping patent holder merges with the non-patent holder, the inequality (27) shows that when there are three producing firms in the market and patent thicket exists (i.e. two firms hold related patents), firms have more incentive to start litigation due to the market competition. To further analyse the impact of merger on litigation risk by comparing the litigation rate in the two cases, I first consider firms' cross-licensing strategy between two patent holders and settlement strategy with the non-patent holder in the following section.

4.2 Ex-ante cross-licensing

For a given credible litigation threat and positive firm values of litigating, the two patent-holding firms can choose whether to form a cross-licensing agreement to avoid litigation. However, this agreement can be used to allocate market shares or conduct other anti-competitive behaviour as a result of collusion. To get rid of antitrust issue, I assume there is no surplus. Moreover, in the ex-ante cross-licensing agreement, firm A and B reach a joint defense agreement that firms would start litigation against firm C or settle simultaneously with equal royalty payment (Choi, 2003; Gilbert, 2002). Similarly, once forming the ex-ante cross-licensing agreement, the two patent holders first decide whether to litigate with the non-patent holder C. If suing C is worthwhile, the two patent holders in the agreement can choose whether to settle with the challenger C and equally split the royalty payment.

Using backward induction, I first analyse two patent holders' litigation or settlement strategies after forming the ex-ante cross-licensing agreement.

Firm C's value of litigating after forming the ex-ante cross-licensing agreement by the two patent holders is the same with the case if no ex-ante cross-licensing agreement, i.e. $V_c^{cl_{sue}} = v_c + (1-P_a)(1-P_b)\pi_c^0 - 2L$. The two patent-holding firms' value if they litigate against firm C together can be written as

$$V_b^{cl_{sue}} = v_b + (1-P_a)(1-P_b)\pi_b^0 + (P_a + P_b - P_aP_b)\pi_b^d - L, \tag{28}$$

$$V_a^{cl_{sue}} = v_b + (1-P_a)(1-P_b)\pi_a^0 + (P_a + P_b - P_aP_b)\pi_a^d - L. \tag{29}$$

The first term of the above two expressions represent the other source of revenues for patent-holding firms, the second term indicates that firm C does not infringe and remain in the market and the third term shows that firm C infringes at least one of the patents owned by the two firms, thus being excluded and two patent holding firms share the duopoly market profits.

The patent-holding firms' values if no litigation against the infringer C can be expressed as

$$V_b^{cl_{nl}} = v_b + \pi_b^0, \tag{30}$$

$$V_a^{cl_{nl}} = v_a + \pi_a^0. \tag{31}$$

Due to the joint defense agreement, two patent holders litigate against firm C if the joint profits in litigation is higher than the joint profit of not litigating, i.e. $V_b^{cl_{sue}} + V_a^{cl_{sue}} \geq V_b^{cl_{nl}} + V_a^{cl_{nl}}$, which gives the litigation against C's condition

$$\mathcal{L}^{cl} = \{(P_a, P_b) | (P_a + P_b - P_a P_b)(\pi^d - \pi_b^0 - \pi_a^0) - 2L \geq 0\}. \tag{32}$$

This condition indicates suing the infringer C is worthwhile for the two patent-holding firms after forming the ex-ante cross-licensing agreement when the joint expected change of profits in litigation is higher than the litigation cost.

When the condition (32) holds, two patent holders can settle with the challenger C. The joint licensing fees paid from the challenger C is r^{cl} and the total settlement fees are $2c$. I assume firm A and B equally split the licensing fees r^{cl} and each pays settlement cost c based on their equal bargaining power.

Firms' values of settlement can be written as

$$V_a^{cl_{settle}} = v_a + \pi_a^0 + \frac{r^{cl}}{2} - c, \tag{33}$$

$$V_b^{cl_{settle}} = v_b + \pi_b^0 + \frac{r^{cl}}{2} - c, \tag{34}$$

$$V_c^{cl_{settle}} = v_c + \pi_c^0 - r^{cl} - 2c. \tag{35}$$

Therefore, the licensing fees can be solved in Nash bargaining,

$$\max_{r^{cl}} [V_c^{cl_{settle}} - V_c^{cl_{sue}}][V_a^{cl_{settle}} + V_b^{cl_{settle}} - V_a^{cl_{sue}} - V_b^{cl_{sue}}],$$

which gives the expression for the licensing fees as follows

$$r^{cl} = \overbrace{(P_b + P_a - P_a P_b)\pi_c^0}^{\text{C's expected value in litigation}} + \frac{1}{2} \underbrace{(P_b + P_a - P_a P_b)(\pi^d - \pi^0)}_{\text{total foregone revenue in judgment}}.$$

Settlement is worthwhile for firms if the joint profit in settlement is higher than the joint profit in litigation, that is

$$\overbrace{4(L-c)}^{\text{total cost savings from settlement}} \geq \underbrace{(P_b + P_a - P_a P_b)(\pi^d - \pi^0)}_{\text{total foregone revenue in judgment}}.$$

and I define the settlement set after forming the ex-ante cross-licensing agreement is $S^{cl} = \{(P_a, P_b) | 4(L-c) - (P_b + P_a - P_a P_b)(\pi^d - \pi^0) \geq 0\}$.

From the above analysis, if $(P_a, P_b) \in \mathcal{L}^{cl} \cap S^{cl}$ holds, two patent holders in the ex-ante cross-licensing agree-

ment chooses to settle with firm C simultaneously. Otherwise, if $(P_a, P_b) \in \mathcal{L}^{cl} \setminus S^{cl}$, two patent-holding firms litigate against firm C simultaneously. I then investigate two patent holders' strategy of whether to form the ex-ante cross-licensing agreement depending on the different litigation or settlement strategy afterwards.

In the cross-licensing agreement, I assume firm B receives a licensing fee r_0[16] from firm A, which is denoted by r_0. However, since $V_i^{cl_{nl}} = V_i^{nl}$, if firms choose to do nothing after the cross-licensing when $(P_a, P_b) \notin \mathcal{L}^{cl}$, the firms' values of forming the ex-ante cross-licensing agreement $V_i^{cl_{nl}} - r_0 - k$ are lower than the values of not litigating V_i^{nl}, therefore, it is not optimal for two patent-holding firms to settle ex-ante and then do nothing. Overall, the two patent-holding firms in a cross-licensing agreement choose whether to settle or litigation the infringer C and based on their strategies, they determine the licensing fee in the cross-licensing agreement.

Therefore, two patent-holding firms' values after forming the ex-ante cross-licensing agreement depending on the litigation or settlement strategy afterwards can be summarized as follows

$$(V_a^{cl}, V_b^{cl}) = \begin{cases} (V_a^{cl_{sue}} - r_0 - k, V_b^{cl_{sue}} + r_0 - k), & \text{for} (P_a, P_b) \in \mathcal{L}^{cl} \setminus S^{cl} \\ (V_a^{cl_{settle}} - r_0 - k, V_b^{cl_{settle}} + r_0 - k). & \text{for} (P_a, P_b) \in \mathcal{L}^{cl} \cap S^{cl} \end{cases} \tag{36}$$

I first determine the licensing fee r_0 in the cross-licensing agreement by solving the maximisation problem

$$max_{r_0} [V_b^{cl} - V_b^{sue}]^{\frac{1}{2}} [V_a^{cl} - V_a^{sue}]^{\frac{1}{2}}.$$

Therefore, the licensing fee r_0 in different cases can be expressed as

$$r_0 = \begin{cases} \frac{1}{2}[(P_b - P_a)\pi^m + (P_b + P_a - 2P_bP_a)(\pi_a^b - \pi_b^d)], & \text{for} \quad (P_a, P_b) \in \mathcal{L}^{cl} \setminus S^{cl} \\ \frac{1}{2}[(P_b - P_a)\pi^m + P_bP_a(\pi_b^d - \pi_a^d) + (P_b + P_a - P_bP_a)(\pi_a^0 - \pi_b^0)]. & \text{for} \quad (P_a, P_b) \in \mathcal{L}^{cl} \cap S^{cl} \end{cases} \tag{37}$$

When two patent holders sue the infringer C simultaneously after forming the cross-licensing agreement (i.e., $(P_a, P_b) \in \mathcal{L}^{cl} \setminus S^{cl}$), the licensing fees between the two patent holders can be written as

$$r_0 = \overbrace{[P_a(1-P_b) + P_b(1-P_a)]\pi_a^d - P_a(1-P_b)\pi^m}^{\text{A's expected change in litigation}} + \frac{1}{2} \underbrace{[P_a(1-P_b) + P_b(1-P_a)](\pi^m - \pi^d)}_{\text{total foregone revenue in judgment}}.$$

In this case, if the duopoly profits earned by firm B and firm C are the same, the licensing fees in forming ex-ante cross-licensing are the same in this case and Case 3 as in Eq(65). This is because, the outside options of forming the ex-ante cross-licensing in both cases are that same, that is the expected profits firms can have through patent litigation, and the expected profits of forming ex-ante settlement in both cases are the duopoly profits.

[16] r_0 can be negative.

Hence, the ex-ante cross-licensing feasible condition can be expressed as

$$\overbrace{2(L^0 - L + P_a P_b k - k)}^{\text{total cost saving from settlement}} \geq \underbrace{[P_a(1-P_b) + P_b(1-P_a)](\pi^m - \pi^d)}_{\text{total foregone revenue in judgment}}.$$

Let $\bar{S}^{sue} = \{(P_a, P_b) | 2(L^0 - L + P_a P_b k - k) - [P_a(1-P_b) + P_b(1-P_a)](\pi^m - \pi^d) \geq 0\}$ to denote the set of (P_a, P_b) where cross-licensing is optimal.

Comparing the above condition $\bar{S}^{sue}$ with the settlement feasibility condition in Case 3 as discussed in Section 3.1.2, I find that the joint expected values in litigation in both cases are the same. However, the cost savings from settlement are different. Since I assume $L^0 - L < L$, the cost savings in this case is smaller than that in Case 3, making ex-ante cross-licensing is less likely.

When two patent holders settle with the infringer C simultaneously after forming the cross-licensing agreement (i.e., $(P_a, P_b) \in \mathcal{L}^{cl} \cap S^{cl}$), the licensing fees between the two patent holders can be written as

$$\begin{aligned} r_0 = & \overbrace{[P_a(1-P_b) + P_b(1-P_a)]\pi_a^0 - P_a(1-P_b)\pi^m - P_a P_b \pi_a^d}^{\text{A's expected change in litigation}} \\ & + \frac{1}{2} \underbrace{[P_a(1-P_b) + P_b(1-P_a)](\pi^m - \pi_a^0 - \pi_b^0) + P_a P_b \pi^d}_{\text{total foregone revenue in judgment}}. \end{aligned}$$

The ex-ante cross-licensing is optimal for both patent holders if the values of ex-ante cross-licensing are higher than the values of litigation, that is

$$\begin{aligned} & \overbrace{2(L^0 - c + P_a P_b k - k)}^{\text{total cost saving from settlement}} \\ & \geq \underbrace{[P_a(1-P_b) + P_b(1-P_a)](\pi^m - \pi^d) + \frac{1}{2}(P_a + P_b - P_a P_b)(\pi^d - \pi^0)}_{\text{total foregone revenue in judgment}}. \end{aligned}$$

Let $\bar{S}^{settle}$ to denote the set of (P_a, P_b) where cross-licensing is optimal.

Compared to the settlement feasibility condition in Section 3.1.2, the above condition in this case shows that first, the cost savings from settlement is lower since firms settle twice before and after forming the cross-licensing agreement. Second, the joint expected revenues in litigation is higher due to the market competition.

Let $\bar{S}$ be the set of (P_a, P_b) for which the above cross-licensing feasibility conditions in two cases hold, which

can be expressed as

$$\bar{S} = \begin{cases} \bar{S}^{sue}, & \text{for} \quad (P_a, P_b) \in \mathcal{L}^{cl} \setminus S^{cl} \\ \bar{S}^{settle}. & \text{for} \quad (P_a, P_b) \in \mathcal{L}^{cl} \cap S^{cl} \end{cases} \tag{38}$$

Overall, for a given credible litigation threat, two patent-holding firms can choose to settle ex-ante or enforce the patent right through patent litigation. Comparing inequalities (66) and (38), it is easy to verify that if the cost of cross-licensing and the gap between the monopoly profit and the overall duopoly market value are the same in two cases, settlement is more likely to occur after the merger because first, the cost saving is larger and second, in a competitive market, the expected revenues from litigation is higher.

The functions of values for firms by considering the counter-sue and cross-licensing can be expressed as follows:

$$V_b^0 = \begin{cases} v_b + \pi_b^0, & \text{if nothing for} \quad (P_a, P_b) \notin \bar{\mathcal{L}} \\ v_b + \pi_b^0 + r_0 - k, & \text{if cross-licensing for} \quad (P_a, P_b) \in \bar{\mathcal{L}} \cap \bar{S} \\ v_b + \pi_b^0 - P_a \pi_b^0 - L^0 & \\ \quad + P_b[(1 - P_a)\pi^0 - \pi_b^0 + P_a(\frac{\pi_c^0}{2} - k)], & \text{if litigation for} \quad (P_a, P_b) \in \bar{\mathcal{L}} \setminus \bar{S} \end{cases} \tag{39}$$

$$V_a^0 = \begin{cases} v_a + \pi_a^0, & \text{if nothing for} \quad (P_a, P_b) \notin \bar{\mathcal{L}} \\ v_a + \pi_a^0 - r_0 - k, & \text{if cross-licensing for} \quad (P_a, P_b) \in \bar{\mathcal{L}} \cap \bar{S} \\ v_a + \pi_a^0 - P_b \pi_a^0 - L^0 & \\ \quad + P_a[(1 - P_b)\pi^0 - \pi_a^0 + P_b(\frac{\pi_c^0}{2} - k)], & \text{if litigation for} \quad (P_a, P_b) \in \bar{\mathcal{L}} \setminus \bar{S} \end{cases} \tag{40}$$

$$V_c^0 = \begin{cases} v_c + \pi_c^0, & \text{if nothing for} \quad (P_a, P_b) \notin \bar{\mathcal{L}} \\ v_c + (1 - P_a)(1 - P_b)\pi_c^0 - 2c, & \text{if settlement for} \quad (P_a, P_b) \in \bar{\mathcal{L}} \cap \bar{S} \\ v_c + (1 - P_a)(1 - P_b)\pi_c^0 - 2L, & \text{if litigation for} \quad (P_a, P_b) \in \bar{\mathcal{L}} \setminus \bar{S} \end{cases} \tag{41}$$

The preceding discussion demonstrates that the firm can resolve patent disputes with other firms through litigation, taking into account the possibility of a counter-sue initiated by the other patent holder, or an ex-ante cross-licensing agreement formed with the other patent holder if I allow firm B to commercialise the patent. Because of the patent thicket, it is more expensive for non-patent holders to settle with patent-holding firms. Furthermore, because of the high revenue in the monopoly market, market competition increases firms' incentive to continue litigation, thus increasing litigation risks. Firm B, however, has an incentive to engage in strategic patenting, i.e., choose not to commercialise the patents and reap the litigation benefits through M&As. This is due to the fact that having sleeping patents, on the one hand, reduces the risk of being sued by the other

patent holder A. Having strong sleeping patents, on the other hand, increases the patent holder's bargaining values in M&As. The aim of this section was not to compare firms' commercialisation with strategic patenting because patent profits are the primary determinant of firms' decisions, but to demonstrate that firms are willing to engage in strategic patenting when the commercial values of patents are low or the patents have not yet been commercialised.

5 Extension

In this section, I extend the analysis into two directions. First, I investigate the impact of patent litigation on R&D[17]. Second, I relax the assumption in the benchmark model that I allow more non-patent holding producing firms.

5.1 Investment in R&D

I analyse firms' R&D decisions in a simplified setting, where I assume away the potential of failure in R&D and firms pay a lump-sum R&D cost, to examine the impact of strategic patenting on firm B and his rival firm A's innovation incentives.

In the initial stage, firms can decide whether to conduct R&D or not depending on the profits and costs of R&D. I have analysed the values after R&D for patent-holding firms A and B in Section 4, which are V_b and V_a [18]. In this section, to establish the relationship between potential patent litigation and R&D, I assume the following relationship between the R&D cost I and firm's win rate P in patent litigation,

$$I_i = \sigma \frac{(P_i)^2}{2},$$

where $i = A, B$ and σ is the scale factor.

The patent-holding firms (i.e. firm A and B) are willing to engage in R&D if firms' values after acquiring the patent is higher than firms' values before conducting R&D. Therefore, for firm B, he is willing to engage in R&D if $V_b - I_b > v_b$, where v_b is firm B's value of other revenues and for firm A, she is willing to engage in R&D if $V_a - I_b > v_a$, where v_a is firm A's value of other revenues. Therefore, if firms can achieve a higher value after R&D, the incentives to innovate becomes stronger. I show that the additional litigation benefits gained through M&A promote firms' innovation incentives in comparative statics (i.e. Section 6).

[17] I do not discuss firm C's infringement decision as this is not the main focus of this paper. Therefore, I assume firm C enters the market in any cases, even if it is not optimal.

[18] Note that the firms' values after R&D (V_i) are based on the fact that both firms A and B are willing to innovate. If only one firm innovates, the value after R&D for the innovator will be higher than V_i as he/she is the only firm holds the patents, while the value for the other party will be lower than V_i. Therefore, if the two firms are not willing to innovate based on V_i, the increased value for the only patent holder may increases firms' incentives to innovate then affect the results. However, the results in this paper are robust because the qualitative results in Section 6 show that there is no region, indicating that both firms do not conduct R&D, so I exclude this possibility.

5.2 Multiple infringers

In this section, I investigate the impact of having multiple infringers on firms' M&A incentives, and examine the impact of market power and degree of market competition on firms' litigation incentives. Multiple infringers are common if they producing similar products that is covered by the same technology in high-tech industry.

In the benchmark model, I focus on the market competition on the patent holders' side as there are two patent holders but only one infringer. However, if there are multiple infringers in the market, how does market competition affect firms' M&A incentives through the channel of patent litigation? To answer this question, I assume there are two patent holders (firm A and firm B) and N potential infringers in the same market. Firm B is the only non-producing firm and can choose either to engage in M&A with the patent holder A to exclude the infringers or merge with the largest challenger firm C in the market to facilitate settlement with A and other infringers. The biggest challenger firm C earns a profit of $\pi_c(\eta + \frac{1-\eta}{N})$ from the products, where N is the total number of challengers and $\eta \in (0,1)$ is C's market power. When $\eta = 1$, the biggest challenger takes all the challengers' market profit of π_c, and when the $\eta = 0$, the biggest challenger shares the remaining market profit equally with other challengers. The overall expected profit of the rest of the challengers is $\pi_c \frac{(1-\eta)(N-1)}{N}$. The profit earned by the patent holder is π_a. The overall market profit for all firms is $\pi^{n+1} = \pi_a + \pi_c$ and the monopoly profit if the patent holder A excludes all infringers is π^m. If there are only two firms in a duopoly market after M&A, firm i earns the profit π_i^d. Since firms are producing similar products (not homogeneous goods), there is no cost reduction for multiple lawsuits and the litigation cost for one lawsuit is fixed, which is denoted by L. To focus on firms' M&A decisions, I also shut down firm B's possibility of commercialisation and proactive patent assertion, i.e. firm B will not litigate against other firms[19]. Firms can choose to settle to save litigation cost and the cost of settlement is $c < L$. For simplicity, I also assume away the possibility of sequential litigation. Therefore, firm A either litigates against all infringers simultaneously or settles with all of them. I still use the Case 2 and 3 to denote the two different outcomes as discussed in Section 3 and Case 1 is the case without M&A.

[19]The Case 1 in the benchmark model is simplified.

5.2.1 Case 1: No M&A

In this simplified Case 1, firm B's value is $V_b^1 = v_b$, which is the revenues not earned from the sleeping patents. With the proof in Appendix A.1.5, firm A and C's values in Case 1 can be expressed as

$$V_a^1 = \begin{cases} v_a + \pi_a + P_a(\pi^m - \pi_a^d) - NL, & \text{if litigation for} \quad P_a > \frac{2N(L-c)}{\pi^m - \pi^{n+1}} \\ v_a + \pi_a + r_1 - Nc, & \text{if settlement for} \quad P_a \in [\frac{NL}{\pi^m - \pi_a}, \frac{2N(L-c)}{\pi^m - \pi^{n+1}}] \\ v_a + v_b + \pi_a. & \text{if nothing for} \quad P_a < \frac{NL}{\pi^m - \pi_a} \end{cases} \tag{42}$$

$$V_c^1 = \begin{cases} v_c + (1 - P_a)\pi_c(\eta + \frac{1-\eta}{N}) - L, & \text{if litigation for} \quad P_a > \frac{2N(L-c)}{\pi^m - \pi^{n+1}} \\ v_c + (\pi_c - r_1^c)(\eta + \frac{1-\eta}{N}) - c, & \text{if settlement for} \quad P_a \in [\frac{NL}{\pi^m - \pi_a}, \frac{2N(L-c)}{\pi^m - \pi^{n+1}}] \\ v_c + \pi_c(\eta + \frac{1-\eta}{N}). & \text{if nothing for} \quad P_a < \frac{NL}{\pi^m - \pi_a} \end{cases} \tag{43}$$

Similar to Case 1 in the benchmark model, merging with another patent holder B increases the overall win rate in litigation, thus increasing the merged firm's likelihood to enforce the patent right through settlement or litigation. Increasing the number of infringers increases the overall litigation costs and decreases A's litigation incentives. However, through M&A, the merged firm can have a higher win rate in court ruling, thus firms have the incentives to merger. Therefore, in the market with multiple infringers, the patent holder A should have a higher incentives to merge than in the benchmark model, where only one infringer exists.

5.2.2 Case 2: M&A between A and B

If merger occurs between the two patent holders A and B in Case 2, the merged firm AB can choose to settle or litigate against all other infringers including the biggest challenger C, which is similar with Case 1 but with higher total win rate in court ruling. Analogous to Case 2 discussed in the benchmark case, the merged firm and firm C's value can be written as

$$V_{ab}^2 = \begin{cases} v_{ab} + \pi_a + P_m(\pi^m - \pi_a^d) - NL, & \text{if litigation for} \quad P_m > \frac{2N(L-c)}{\pi^m - \pi^{n+1}} \\ v_{ab} + \pi_a + r_2 - Nc, & \text{if settlement for} \quad P_m \in [\frac{NL}{\pi^m - \pi_a}, \frac{2N(L-c)}{\pi^m - \pi^{n+1}}] \\ v_{ab} + \pi_a. & \text{if nothing for} \quad P_m < \frac{L}{\pi_c(\eta + \frac{1-\eta}{N})} \end{cases} \tag{44}$$

$$V_c^2 = \begin{cases} v_c + (1 - P_m)\pi_c(\eta + \frac{1-\eta}{N}) - L, & \text{if litigation for} \quad P_m > \frac{2N(L-c)}{\pi^m - \pi^{n+1}} \\ v_c + (\eta + \frac{1-\eta}{N})(\pi_c - r_2) - c, & \text{if settlement all for} \quad P_m \in [\frac{NL}{\pi^m - \pi_a}, \frac{2N(L-c)}{\pi^m - \pi^{n+1}}] \\ v_c + \pi_c(\eta + \frac{1-\eta}{N}). & \text{if nothing for} \quad P_m < \frac{L}{\pi_c(\eta + \frac{1-\eta}{N})} \end{cases} \tag{45}$$

where $P_m = P_a + P_b(1 - P_a)$ and $r_2 = P_m\pi_c + \frac{1}{2}(\pi^m - \pi^{n+1})$

5.2.3 Case 3: M&A between B and C

I then analyse how merged firm BC solve the patent disputes with firm A in Case 3. With the proof in Appendix A.1.6, the merged firm BC's value has a similar form with the firm value in Section 4, that can be written as

$$V_{bc}^3 = \begin{cases} v_{bc} + \pi_c(\eta + \frac{1-\eta}{N}), & \text{if nothing for} \quad (P_a, P_b) \notin \bar{\mathcal{L}}^n \\ v_{bc} + \pi_c(\eta + \frac{1-\eta}{N}) + r^n - k, & \text{if settlement for} \quad (P_a, P_b) \in \bar{\mathcal{L}}^n \cap \bar{S}^n \\ v_{bc} + (1-P_a)(1-P_b)\pi_c(\eta + \frac{1-\eta}{N}) & \\ +P_aP_b(\pi_c^d - k) + P_b(1-P_a)\pi^m - NL, & \text{if litigation for} \quad (P_a, P_b) \in \bar{\mathcal{L}}^n \setminus \bar{S}^n \end{cases} \tag{46}$$

$$V_a^3 = \begin{cases} v_a + \pi_a, & \text{if nothing for} \quad (P_a, P_b) \notin \bar{\mathcal{L}}^n \\ v_a + \pi_a - r^n - k, & \text{if settlement for} \quad (P_a, P_b) \in \bar{\mathcal{L}}^n \cap \bar{S}^n \\ v_a + (1-P_a)(1-P_b)\pi_a & \\ +P_aP_b(\pi_a^d - k) + P_a(1-P_b)\pi^m - NL, & \text{if litigation for} \quad (P_a, P_b) \in \bar{\mathcal{L}}^n \setminus \bar{S}^n \end{cases} \tag{47}$$

where

$$r^n = \begin{cases} \frac{1}{2}[(P_b - P_a)\pi^m + (P_b + P_a - 2P_bP_a)(\pi_a^b - \pi_{bc}^d)], & \text{for} \quad (P_a, P_b) \in \mathcal{L}_m^{cl} \setminus S_m^{cl} \\ \frac{1}{2}[(P_b - P_a)\pi^m + P_bP_a(\pi_{bc}^d - \pi_a^d) + (P_b + P_a - P_bP_a)(\pi_a - \pi_c(\eta + \frac{1-\eta}{N}))], & \text{for} \quad (P_a, P_b) \in \mathcal{L}_m^{cl} \cap S_m^{cl} \end{cases}$$

where $\bar{S}^n$ denotes the overall ex-ante cross-licensing condition as expressed in inequalities (115) and (116) and in Appendix A.1.6. $\bar{\mathcal{L}}^n$ is the litigation condition between two patent holders, S_m^{cl} is the settlement feasibility condition and $\mathcal{L}_m^{cl}$ is the litigation condition with other infringers after cross-licensing, which are all defined in Appendix A.1.6.

Follow the same four steps in the simplified three-party bargaining game discussed in Section 3.3, I obtain firm B's bargaining values in Case 2 and Case 3 and firm B makes his M&A decisions by comparing the values in different cases. To examine the impact of market competition on the infringers' side on firms' M&A incentives, I further show the M&A outcomes qualitatively in Section 6.

6 Comparative statics

In this section, I numerically analyse how the M&A target choice, the commercialisation, and the R&D decision depend on the parameters of the model. The base case parameter values are given in Table 1. In the base case, I assume firms' other revenues that are unrelated with the patents at the initial stage are the same (i.e., $v_a = v_b =$

$v_c = 10$)[20] and the profits earned from the homogeneous products are the same for firm A and C (i.e., $\pi_a^d = \pi_c^d = 4$)[21] when firm B is not in the market. The monopoly profit is 10 (i.e. $\pi^m = 10$). When firm B produces, firms' profits are $\pi_a^0 = \pi_b^0 = \pi_c^0 = 2$.[22] The M&A costs are higher than settlement costs but lower than the litigation costs[23], and the cost of cross-licensing is lower than the cost of settlements.

I mainly examine how the strength of patent rights, as reflected by P_a and P_b, influence firm decisions including patent litigation strategies, M&A, commercialisation and R&D in my model, as well as how market competition affects firms' M&A incentives. I demonstrate the robustness test further by varying the costs and profits associated with M&As. The results plotted in Figure 6 to Figure **??** are based on the base case parameters with varying win rates to investigate the impact of patent litigation on firms' innovation, commercialisation and M&A decisions. To show that firms' M&A incentives are not only driven from the incentives for cost saving, I show the firms' decisions of commercialisation and M&A by changing the M&A costs, which can also show the impact on M&A incidence by considering other synergies in profits. I further analyse firms' strategic patenting incentives by shutting down the possibility of holding sleeping patents. I also change the profits and costs from the base case to investigate the impact of market competition and show the comparative statics results.

[Insert Table 1 here.]

6.1 Firms' decisions in the base case

In order to understand how are patent litigation related to the incidence of M&A activities, I show the patent litigation outcome, i.e, nothing, settlement or litigation under different cases in Figure 6. The four graphs show the litigation and settlement regions with and without M&A with different P_b and P_a assuming R&D takes place in the whole region when (I) no merger with sleeping patents owned by firm B (i.e. Case 1), (II) merging between two patent holders A and B with sleeping patents (i.e. Case 2), (III) merging between non-producing patent-holding firm B and non-patent holder C (i.e. Case 3), and (IV) two producing patent holders and no M&A. The lines represent the boundaries of different litigation outcomes. The region $Settle_{A\&B}$ or $Litigate_{A\&B}$ in panel (II) represent the region where the merged firm AB settle or litigate with C. In $Settle_A$ region, firm A obtain the royalty payment in settlement. Vice versa, in $Settle_B$ region, firm B gains a royalty payment in settlement. In panel (III) and (IV), settlement takes the form of cross-licensing. $Settle^*$ indicates that two patent holders cross-license ex-ante but sue the challenger C in the next stage in panel (IV). In region $Litigate$, firms go through the patent litigation without settlement. In NIL (shorthand for No Incentive to Litigate), no litigation or settlement occurs.

[20] Other revenues should be high enough to cover all costs associated with litigation, settlement or M&A.
[21] This is possible if firm C is a perfect imitator.
[22] I assume equal profits and other revenues to eliminate the effect of pre-litigation and pre-merger firm value on firms' decisions.
[23] The main results of firms' M&A incentives are robust even if the M&A costs are higher than the litigation costs.

[Insert Figure 6 here.]

The litigation and settlement equilibrium depend on firms' M&A and commercialisation decisions. In Panel (I), when no M&A, firm A initiates litigation first by considering the licensing fees paid to firm B in the next stage. When both P_a and P_b are low enough, there is no credible litigation threat as in NIL region. The patent holder A starts litigation when P_a is high and P_b is low (see $Litigate_A$ region). This is because, first, when P_a is high, firm A has a strong incentive to gain the monopoly profit through litigation. Second, by taking into account the fact that P_b is too low to obtain the licensing fees for firm B, firm A has more incentive to start patent litigation since the profit in litigation is high enough to give up settlement. However, firm A's litigation incentive is lower When P_b decreases and firm A thus settle with firm C as in $Settle_A$ region. However, in $Settle_B$ region, firm A does not litigate to against firm C but need to pay licensing fee to the other patent holder B. In particular, when P_b is in between 0.24 and 0.6, firm B is able to settle with firm A once A is the monopolist through patent litigation. Knowing this, firm A has less incentive to start litigation. In general, the figure shows a large settlement region and a small litigation region, which implies the existence of another patent holder B reduces the market litigation risk.

Panel (II) shows the litigation outcomes when the merger occurs between two patent holders (i.e. Case 2), the merged firm can file the patent infringement lawsuit based on a larger patent portfolio, thus increasing the win rate. The likelihood of litigation increases significantly compared to Case 1 in Panel (I). This figure also explains why firms have an incentive to accumulate patents even with no commercialisation. On the one hand, a large patent portfolio protects the current products and increase the likelihood of gaining monopoly profit through patent litigation. On the other hand, even if the win rate based on a single patent is not high enough, the large patent portfolio facilitates settlement and allow firms to obtain licensing revenues.

Panel (III) shows the litigation outcomes in Case 3 where non-producing patent-holding firm B engages in M&A with the non-patent holder C. In this case, two competitors in the market both own the related patents and can sue each other. They can also form the cross-licensing agreement when the threat of litigation is credible and settlement is worthwhile for both parties. The figure shows that settlement occurs when one of the firms has a sufficiently high win rate, while the other has a sufficiently low win rate in patent litigation. This is because firms' willingness to settle (and pay licensing fees) are high when each firm knows his rival has a high win rate in the lawsuit. Specifically, when P_B is relatively higher, firm A is willing to pay licensing fee to form cross-licensing. Otherwise, firm B pays licensing fee when P_A is relatively higher. However, when the win rate for one firm is too low to threaten its rival with litigation, litigation occurs. One striking feature is that when both P_A and P_B are both high, unlike the litigation outcomes in Panel (I) and (II), there is no credible litigation threat because both firms know they will lose if the other party sues, resulting in the no litigation equilibrium.

The last Panel (IV) shows the litigation and settlement outcomes if all firms A,B and C are producing in the

same market. M&As thus are not allowed between any of the two firms. Firm A and B can still reach the cross-licensing agreement in this case. However, compared to Panel (III), the regions of ex-ante cross-licensing shrinks, which indicates that the likelihood of settlement drops significantly. The comparison of Panel (III) and (IV) shows that the market competition increases the likelihood of litigation, especially when win rates are not high enough for firms. Morton (2012) raises the empirical puzzle that competitors frequently sue each other instead of keeping the "patent peace". This model shows a consistent result as in Choi and Gerlach (2017) who theoretically show the positive impact of market competition on litigation risk by considering the possibility of cross-licensing. I further show the patent thicket, which is the fragmented patent ownership, in intensive product market competition also leads to substantial patent litigation risk. In addition to this, merger internalizes the externalities that allow for patent hold up between patent holders and challengers, thus facilitates cross-licensing (Dorsey, 2013).

Next, I summarise the different M&A outcomes and the settlement or litigation outcomes at the base case with varying win rates for two patent-holding firms in Figure 7, assuming firm B holds sleeping patents in the whole region. The blank area is the region where no M&A (i.e. Case 1), and the blue area is the region where M&A occurs between two patent-holding firms A and B (i.e. Case 2) and green area is the region where M&A occurs between the non-producing patent holder B and the producing firm C that has no patent (i.e. Case 3). The figure shows that M&A in both Case 2 and 3 can occur in the base case and the win rates for both patent holders affect firms' M&A decisions.

[Insert Figure 7 here.]

Even I assume away any synergies associated with producing profits in M&A, firms are still willing to to engage in M&A due to the additional values gained in patent litigation through M&A. In general, the likelihood of M&A increases as P_b increases. This is because firms can extract litigation benefits based on strong patent protection. When P_b is high, firm B is able to enforce its patent rights through patent litigation or settlement, thus it increases other firms' incentives to engage in M&A with firm B to reduce litigation risks or gain the competitive advantages through patent litigation.

Furthermore, the M&A target for firm B can either be a patent-holding firm in Case 2 or an alleged infringer in Case 3 depending on P_b and P_a. Firstly, two patent holders merge in the top right corner of this figure. When both P_a and P_b is high, both patent-holding firms know neither of them are not likely to exclude each other via patents lawsuits but they can exclude the non-patent holder C to gain the monopoly profit and save the joint litigation costs by merging. This result is consistent with the empirical evidence in Marco and Rausser (2008) who empirically find that firms with overlapping technologies and strong patent rights tend to merge in order to reduce mutually blocking technology. Secondly, in the lower right corner of the figure, the non-producing patent holder B merges with the firm C in Case 3. This is because when P_a is low and P_b is high, merging allows firm B to gain the

possibility of excluding the other patent holder with lower win rate and allow firm C to stay in the market, which increasing both parties' M&A incentives. Thirdly, when the P_b is in the middle range, the litigation threat based on firm B's patent is not strong enough, therefore, firm C has less incentives to engage in M&A. However, it is still worthwhile for firm B to merge with firm A because of the increased win rate of the combined firm AB against firm C. Finally, when P_b is low, merging is less attractive for the patent holder A but it is worth merging with the C to reduce the litigation risk.

The conventional wisdom argues firms choose to merge with firms with special assets such as patents (Kogut and Zander, 1992; Nelson and Winter, 1997). However, this model shows the impact of patent litigation on firms' M&A decisions and thus merger target can be a non-patent holder[24]. The final litigation outcome is a result of bargaining based on different litigation outcome as discussed in Section 3.3. Each firm will make the M&A decision by taking into account the possibility of merger between firm B with his rival. Therefore, firms' incentives to engage in M&A are not only affected by their own patent litigation outcomes after merger but also patent litigation outcomes after merger for the other party. This "strategic M&A" has also been studied in the M&A literature but this is the first study to discuss it from the perspective of patent litigation (Gupta, 2012).

I also find that assuming R&D has taken place in the whole parameter region, firm B chooses to commercialise the patents only if P_a is low enough (The figure is not presented for brevity). This lends credence to the argument that patent litigation increases the likelihood of M&A activity. When P_a is low, the patent litigation initiated by firm A does not pose a significant threat to firm B if he produces, so the firm is willing to enter the market through commercialisation. Otherwise, it is beneficial for firm B to sleep patents in order to avoid being sued and gain additional value through M&As. Therefore, firm B has the incentives to engage in strategic patenting to avoid patent litigation.

6.2 Strategic patenting

In this section, I investigate how patent litigation affects firms' incentives to conduct R&D, particularly the incentives to engage in strategic patenting.

Figure 8 shows firms' R&D region in the base case and the case assuming firm B commercialises his patents instead of sleeping them. The blank region is the area where both firms are willing to innovate, where firms' values after R&D are higher then the value of no R&D. The dotted lines shaded area shows firm B has no incentive to innovate, i.e., firm B's value after R&D is too low to afford the R&D cost, and the solid lines shade region represents firm A has no incentive to innovate, i.e., firm A's value after R&D is too low to afford the R&D cost. Both figures show that a patent-holding firm is not willing to engage in R&D when his win rate is low but the rival's win rate is high due to the possibility of being sued by the rival. This also indicates in industries where the

[24]The robustness check shows that even if I separate M&A and patent acquisition, Case 3 still occurs.

boundary of patents is not clear and litigation risk is high due to the weak patent protection, firms' incentive to innovate decreases.

[Insert Figure 8 here.]

By comparing the two figures, I find the absence of sleeping patents reduces both firms' incentives to innovate. Firm B, which has the option of commercialisation or strategic patenting, has an incentive to engage in strategic patenting because he can benefit from M&A even when not producing. On the one hand, the commercialised patent holder A can increase her profits by merging with the sleeping patent holder. However, even if the merger takes place between the non-patent holder and the sleeping patent holder, the merger increases the likelihood of ex-ante settlement between the two patent-holding firms, thus reducing the costs of resolving patent dispute and increasing firm A's incentives to innovate. Therefore, a merger increases the outsider's incentive to innovate in less innovative technologies (i.e. low P).

6.3 Market competition

In this section, I examine the impact of market competition on firms' M&A incentives and commercialisation decisions when R&D is assumed to have occurred in the whole region. The value difference in a monopoly market (π^m) and a duopoly (π^d) or three firms' market (π^0) measures the degree of market competition, i.e., the level of market competition is captured by $\pi^m - \pi^d$ or $\pi^m - \pi^0$.

Figure 9 depicts firms' M&A decisions with varying P_a and P_b. The Panel (I) shows the M&A outcomes in the base case, Panel (II) shows the outcomes when the monopoly profit is increased from 10 to 13 while keeping all other parameter values to the base case, Panel (III) demonstrate the M&A decisions when the overall duopoly profit is higher than that in the base case, and the last Panel (IV) shows the outcomes when the overall profit in the three firms' market is higher. By measuring the market competition in the model, the level of market competition is highest in Panel (II), then decreases from Panel (I) to (III) and finally becomes the lowest in Panel (IV). Overall, changes in profits and costs in patent litigation influence M&A decisions, and market competition increases the possibility of merging with a non-patent holder due to the increased litigation risk.

[Insert Figure 9 here.]

When there is fierce market competition (the value difference is larger) as shown in the Panel (II), the likelihood of a merger between firm B and firm C (i.e. Case 3) increases. This is because, firstly, when the market competition is intense, the patent holder A who competes in the market has a strong incentive to force the rivals out of the market and the litigation condition can be satisfied even with low P_a, therefore lowering her incentives to merge, especially when P_b is not high enough. Secondly, the non-patent holder C has a strong incentive to merge with

the sleeping patent holder to reduce the litigation risks. When the monopoly profit is high, the likelihood of settlement is reduced in two M&A cases (i.e. Case 2 and 3) according to the settlement conditions in Eq(56) and Eq(66). However, when two patent holders merge, the likelihood of a settlement decreases more. As a result, litigation occurs in a wider range of parameter set as in Panel (II) than in the base case, which increasing firm C's incentive to merge in order to stay in the market.[25] Therefore, firm C has a strong M&A incentive when P_b is high.

However, when market competition is limited, as shown in Panel (III), firm C's M&A incentives are reduced, lowering the possibility of Case 3. Moreover, the difference between monopoly profit and three-firm market profits has minor impact on M&A by comparing Panel (IV) and (I). This is because, profits in patent litigation with and without M&A are equal to those in the base case.

I further investigate the impact of market competition on firms' M&A incentives if there are multiple infringers in the market with asymmetric market power and the win rates for all associated patents are 0.5, i.e. $P_a = P_b = 0.5$. Figure 10 shows the M&A outcomes in the case when two patent-holding firms merge (i.e. Case 2) and the case when the sleeping patent holder merges with the biggest challenger (i.e. Case 3). The figure shows that the sleeping patent holder is more willing to merge with the non-patent holder when both the number of infringers N and the biggest challenger's market power η are high. This is because by merging with the sleeping patent holder, the biggest challenger can gain an additional values in patent litigation against the patent holder A and other infringers. On the one hand, the biggest challenger has a higher incentive to remain in the market due to the high market profit with his high market power. Through M&A, the litigation risk from patent holder A is reduced. On the other hand, the biggest challenger can gain additional profits from other infringers with the credible litigation threat through M&A. The higher the number of infringers in the market, the more benefits he can obtain. This result indicates that in a competitive market (i.e. N is high), if there is a non-patent holder with strong market power, he is likely to accumulate patents through M&As.

[Insert Figure 10 here.]

7 Discussions

In this section, I discuss the main modelling assumptions and the potential limitations and implications of these assumptions.

First, in this paper, I impose a strict antitrust restriction that competitors cannot engage in M&A and I assume away antitrust concerns in settlement or cross-licensing. The recent papers reconcile the conflicts of antitrust in patent system by examining firms' innovation incentives and outputs (Carrier, 2002; Day, 2017; Barton, 1996; Cheng, 2012). This paper provides a model to study the impact of patent litigation on M&A and firms' innovation

[25] I further examine the M&A regions with different monopoly profit π^m and M&A costs c^m and the results are consistent.

incentives, and the results in my model suggest that firms have stronger innovation incentives when M&As occur. As a result, I expect that my results will be robust when considering the balance of patent-antitrust paradox.

Second, I assume patent originality (i.e. win rates) and patent quality (profits earned from the patent) are independent. This is a standard assumption in patent litigation literature. For example, Norbäck et al. (2020) investigates the impact of patent quality and win rate on firms' M&A choices separately and find firms choose to acquire and sleep low quality patents. However, there is likely to have a positive correlation between originality and quality, that is, when profit is large, the win rate is also large. Incorporating this possibility would significantly complicate the model and can be an extension in future research.

Third, I do not explicitly model non-producing entities (NPEs) in this model, though firm B with sleeping patents acts like an NPE when M&A is infeasible. In this case, the cross-licensing is less likely without the threat of counter-litigation. However, this model shows the benefits of having sleeping patents for defensive purpose after M&A. This defensive patenting strategy is less likely for NPEs, though Fischer and Henkel (2012) and Haber and Werfel (2016) find NPEs can help small innovators to protect their patent rights. This model can be used to the examine NPE litigation if I remove other revenues. However, the severe financial constraints caused by patent litigation may preclude M&A activity.

Finally, I do not model the patent hold-up problems in this paper because all royalty payments are determined in Nash bargaining with equal bargaining power. However, merger is a way to solve the patent hold-up problem if the bargaining power of the patent holder is too high to set a reasonable royalty payment because merging increases the overall bargaining power and can change the patent litigation outcomes. Therefore, The licensee with low bargaining power can merge with a third party who has sleeping patents to gain a better bargaining position in licensing negotiation as discussed in Section 3.1.2. This is left for future research.

8 Conclusion

I develop a static multi-stage framework for three parties: a patent holder, a non-patent holder, and a third parties who holds sleeping and related patents and can merge with either the patent holder or the non-patent holder to resolve patent disputes. From this model, I uncover a specific source of synergy - patent litigation benefits - that drives M&As. I show that M&A occurs when the winning probability of sleeping patents in patent litigation is high enough because the combined firm can gain litigation benefits using the strong patents. In addition, patent litigation influences firms' M&A target selection. When two patent-holding firms' patents are both strong enough to exclude others, they are more likely to engage in M&A, whereas the non-patent holder can be chosen as the M&A target when the possibility of invalidating the other patents is high. Moreover, the degree of market competition increases patent holders' incentives to gain monopoly profits through patent litigation, which leads to stronger

incentives for non-patent holders to engage in M&As due to the high litigation risk. This paper also discovers that patent-holding firms commercialise patents when the win rate of other patents is low, thereby avoiding the litigation risk. I establish the relationship between patent litigation and corporate innovation in this paper. I first show the synergies in M&A resulting from patent litigation increase firms' incentives to engage in strategic patenting theoretically. Furthermore, strategic patenting does not reduce innovation incentives for other competitors.

The findings of this paper suggest the following new research directions for the future. First, this paper shows how patent litigation based on sleeping patents can drive merger transactions, demonstrating the empirical implication that M&A synergies can be obtained in areas other than product markets and technological innovations. Second, I show the win rate of sleeping patents plays an important role in determining M&A target choice and innovation decisions. This model can be extended to examine how divergent expectations of the subjective probability of winning in patent litigation for two parties affect the M&As and pre-merger innovations under information asymmetry and managerial optimism. Third, I assume a equal bargaining power between any of two firms to simplify the model. Asymmetric bargaining power can be included into future work as firms' bargaining power are affected by firms' financial conditions, which are related to the financial constraints caused by patent litigation. Finally, while this model captures patent relatedness, patent litigation win rate, and patent profits, it does not construct the relationships between them. Future theoretical work is required to consider competition due to product market rivalry as well as complementarities due to overlapping technologies.

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Table 1: Parameters in the Base Case

Parameter	**Value**
Firms' value of other revenues	$v_a = v_b = v_c = 10$
Monopoly profit	$\pi^m = 10$
Profits earned from the homogeneous good	$\pi_a^d = \pi_c^d = 4, \pi_a^0 = \pi_b^0 = \pi_c^0 = 2$
Litigation cost for each firm	$L = 1.2$
Total litigation cost for multiple infringers	$L^0 = 2$
M&A cost for each firm	$c^m = 0.7$
Settlement cost for each firm	$c = 0.5$
Joint cost in cross-licensing	$k = 0.4$
Scale factor in R&D	$\sigma = 0.7$

Figure 1: Three Possible M&A Strategies for Non-Producing Patent Holder ("B")

The three shows three possible M&A strategies for firm B. The Producing Patent Holder is represent by "A" and the Non-Patent Holder is represented by "C"

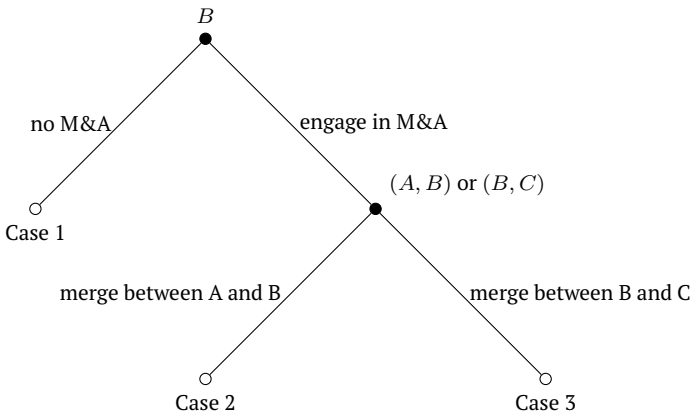


Figure 2: Strategies and Payoffs of the Merged Patent Holder ("AB") and the Non-Patent Holder ("C") In Case 2

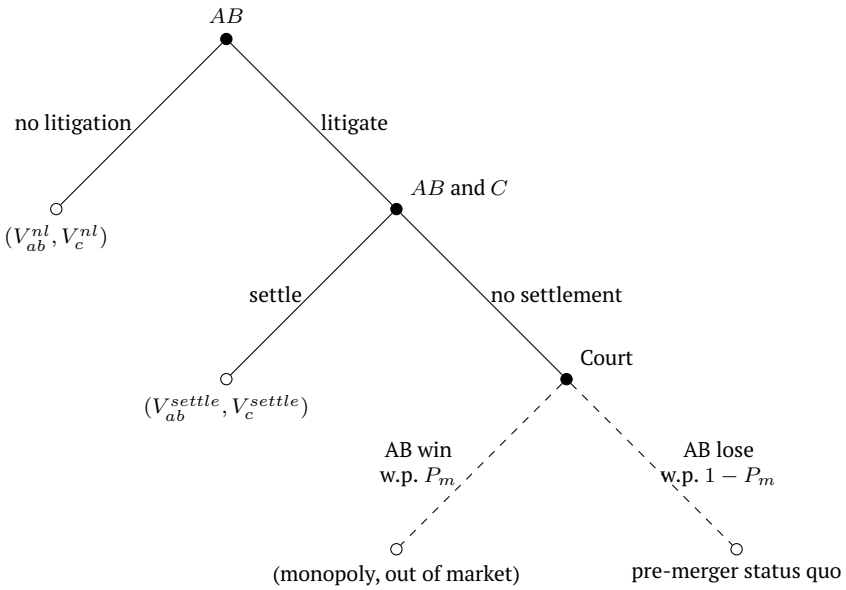


Figure 3: Strategies and Payoffs of the Merged Firm ("BC") and the Other Patent Holder ("A") In Case 3

This figure presents the decision tree for the merged firm in Case 3 when M&A occurs between the non-producing patent holder B and the firm C that has patent. In this case, two firms in the market both hold patents. Since I assume no additional litigation cost for counter-claim, firms start litigation by considering the possibility of counter-sue and thus both firms have symmetric decision trees. Initially, the merged firm decides whether it is worthwhile to litigate by considering the counter-sue initiated by other patent holder A. With credible litigation threat, firms can cross-license (which is denoted by "CL") each other's patents ex-ante to save costs. If litigation continues, the merged firm gains monopoly profit only if he wins in two suits with probability $P_b(1-P_a)$. Otherwise, he cross-license ex-post when both firms win with probability P_bP_a, share the market when both firms lose with probability $(1-P_b)(1-P_a)$ or being excuded if he loses and the other party wins with probability $(1-P_b)P_a$.

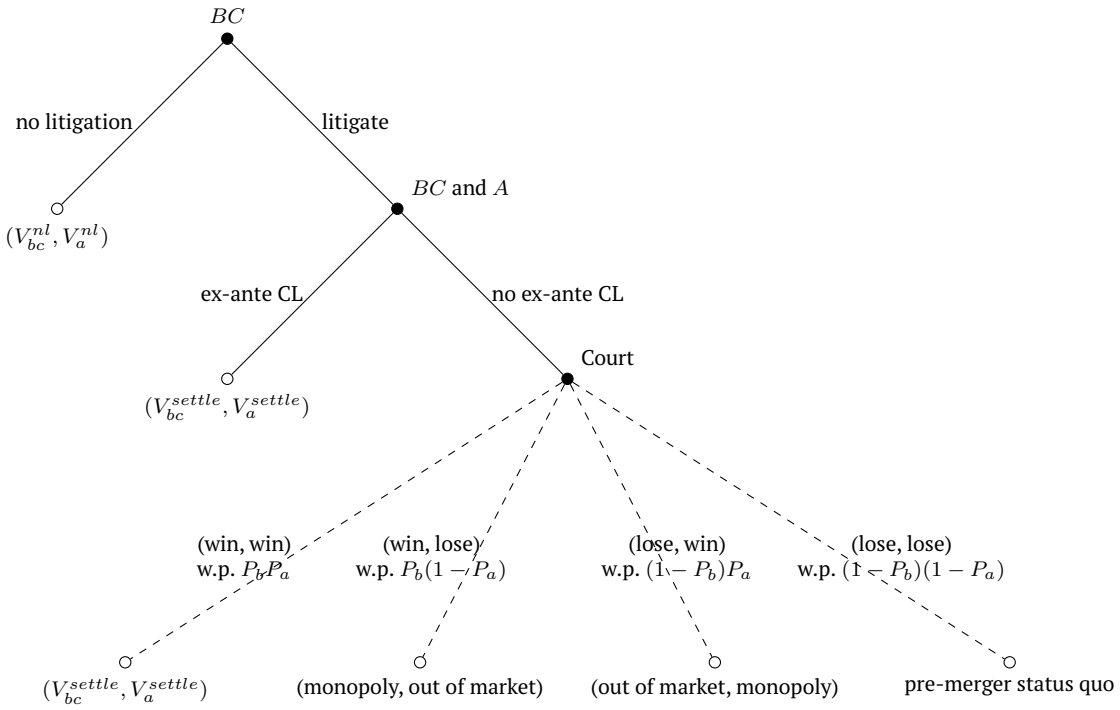


Figure 4: Strategies and Payoffs of the Patent Holder ("a"), the Non-Patent Holder ("C") and the Bidder with Sleeping Patents ("B") When No M&A in Case 1

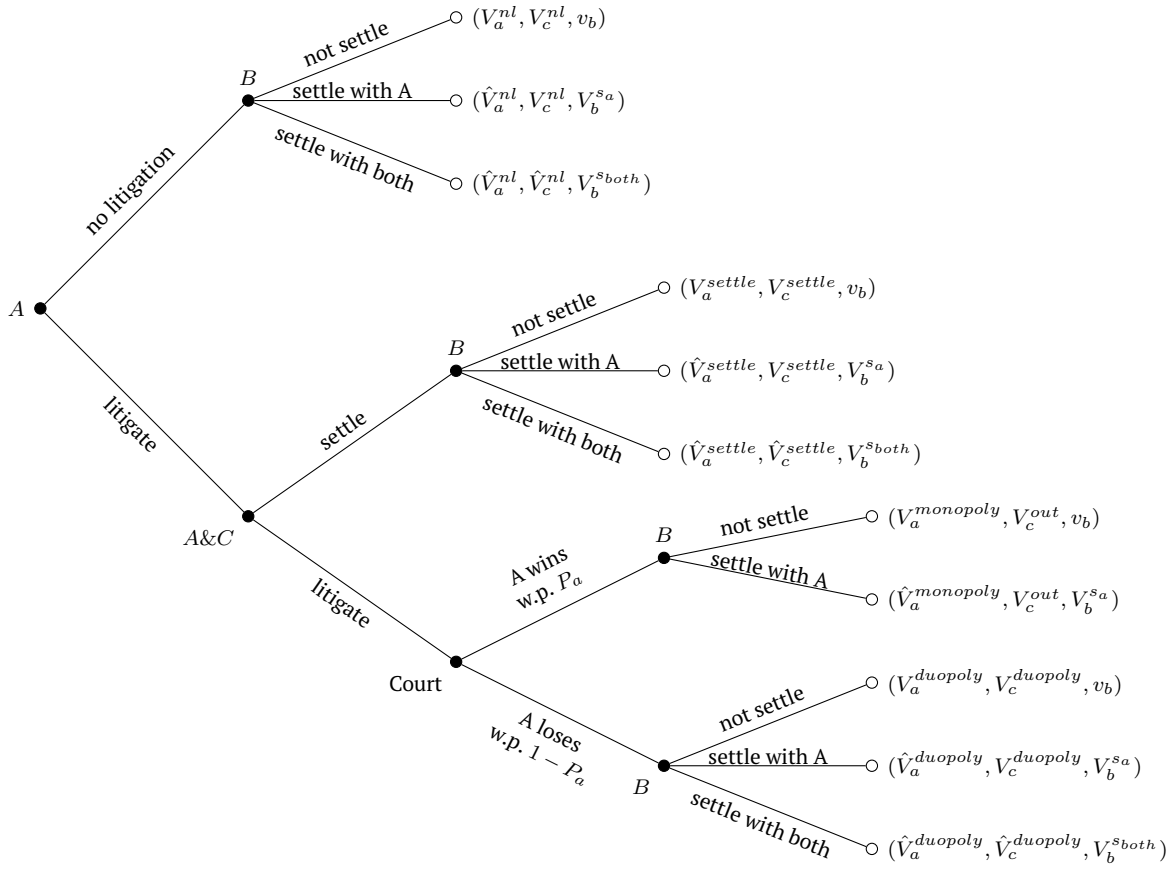


Figure 5: Sequence of Events after R&D

This figure presents the sequence of events after R&D by patent holders. Initially, I assume firm A will commercialise the patent and compete with a non-patent holder C, firm B decides whether to join the competition through commercialisation or keep the patent asleep. If firm B holds sleeping patents, M&A occurs and the outcome is Case 1, 2 or 3. Otherwise, based on the strict antitrust law, M&A is not allowed. Firms resolve their patent disputes through litigation or settlement. Two patent holders can cross-license (CL) with each other and then decide whether to litigate or settle with C simultaneously.

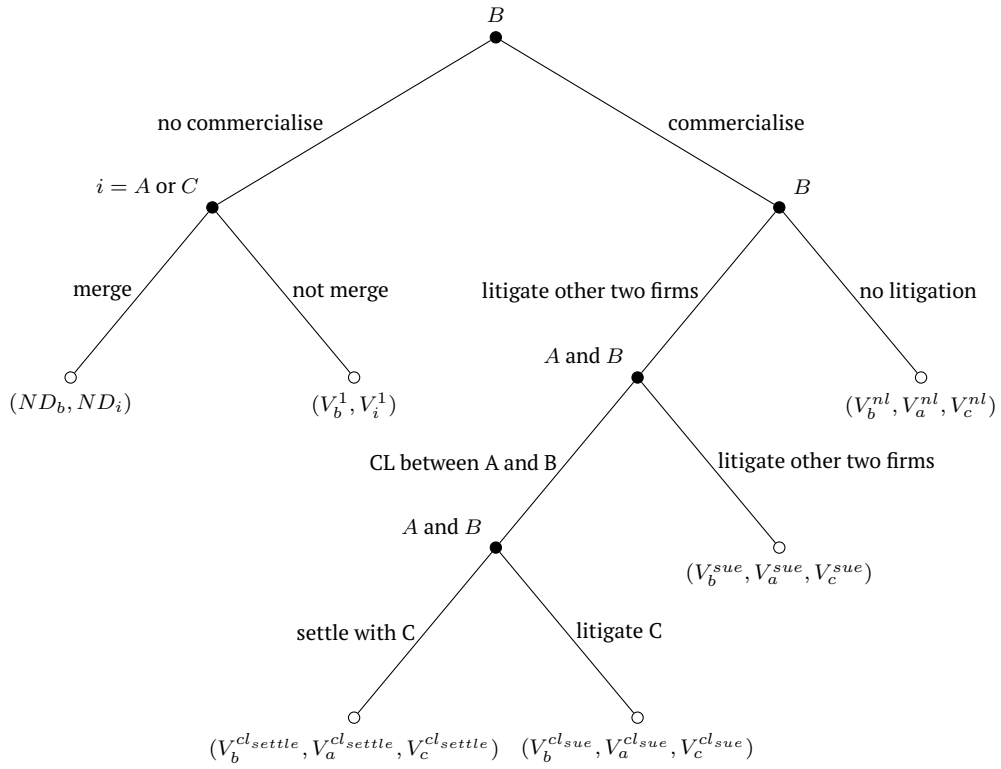


Figure 6: Litigation Outcomes in Four Cases with Win Rate P_b and Win Rate P_a

The four graphs show the litigation and settlements with and without M&A when (I) no merger with sleeping patents owned by firm B (i.e. Case 1), (II) merging between two patent holders A and B with sleeping patents (i.e. Case 2), (III) merging between non-producing patent-holding firm B and non-patent holder C (i.e. Case 3), (IV) two producing patent holders and no M&A, with different P_b and P_a assuming R&D takes place in the whole region. The lines represent the boundaries of different litigation outcomes. The region $Settle_{A\&B}$ in panel (II) represents the region where the threat of litigation is credible for both firm A and B to facilitate settlement out of court. In $Settle_A$ region, firm A obtain the royalty payment in settlement. Vice versa, in $Settle_B$ region, firm B gains a royalty payment in settlement. In panel (III) and (IV), settlement takes the form of cross-licensing. $Settle^*$ indicates that two patent holders cross-license ex-ante but sue C in the next stage in panel (IV). In region $Litigate$ or L, firms go through the patent litigation without settlement. In NIL (shorthand for No Incentive to Litigate), no litigation occurs for both patents.

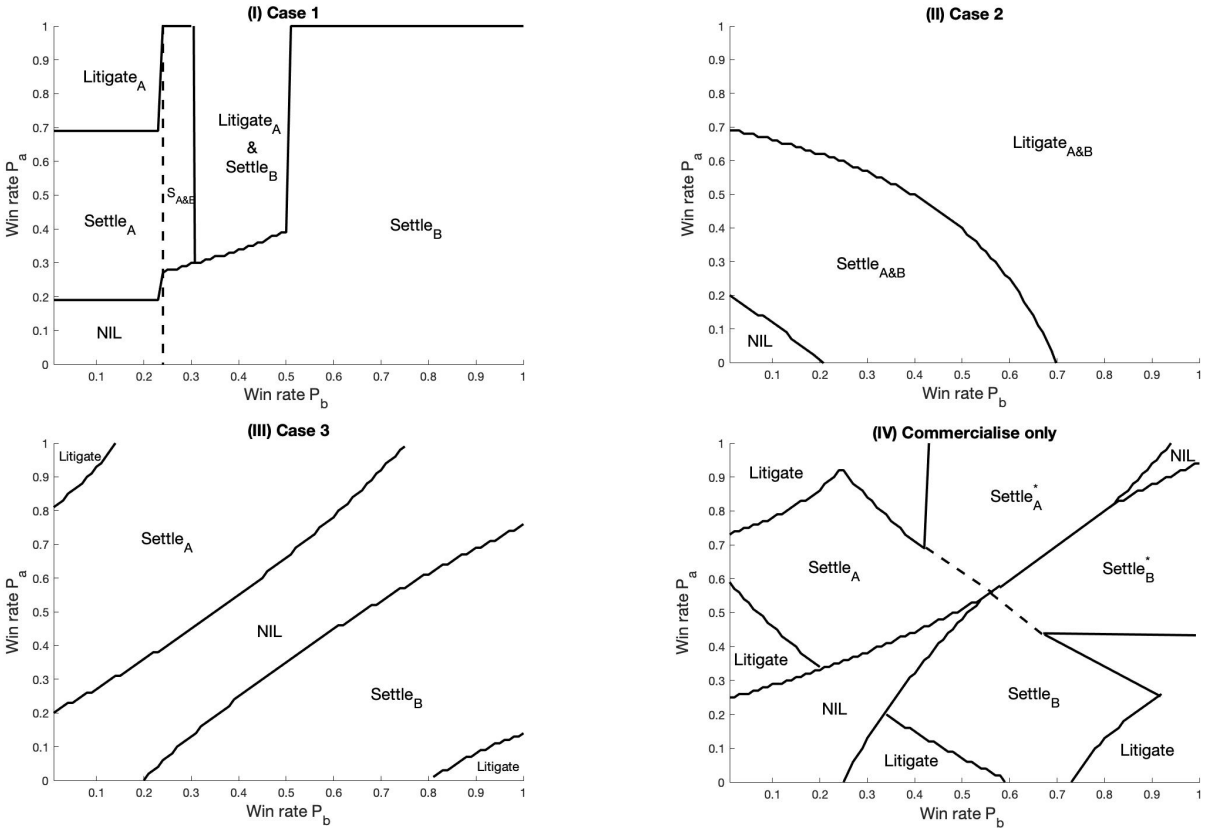


Figure 7: Firms' M&A Outcomes with Win Rate P_b and Win Rate P_a

The graph shows the different M&A outcomes and the settlement or litigation regions in the base case assuming firm B holds sleeping patents in the whole parameter set. The blue area is the region where two patent-holding firms A and B merge (i.e. Case 2) and the green area is the region where the sleeping-patent holder B and the non-patent holder C merge (i.e. Case 3). The blank region is the area where there is no M&A (i.e. Case 1).

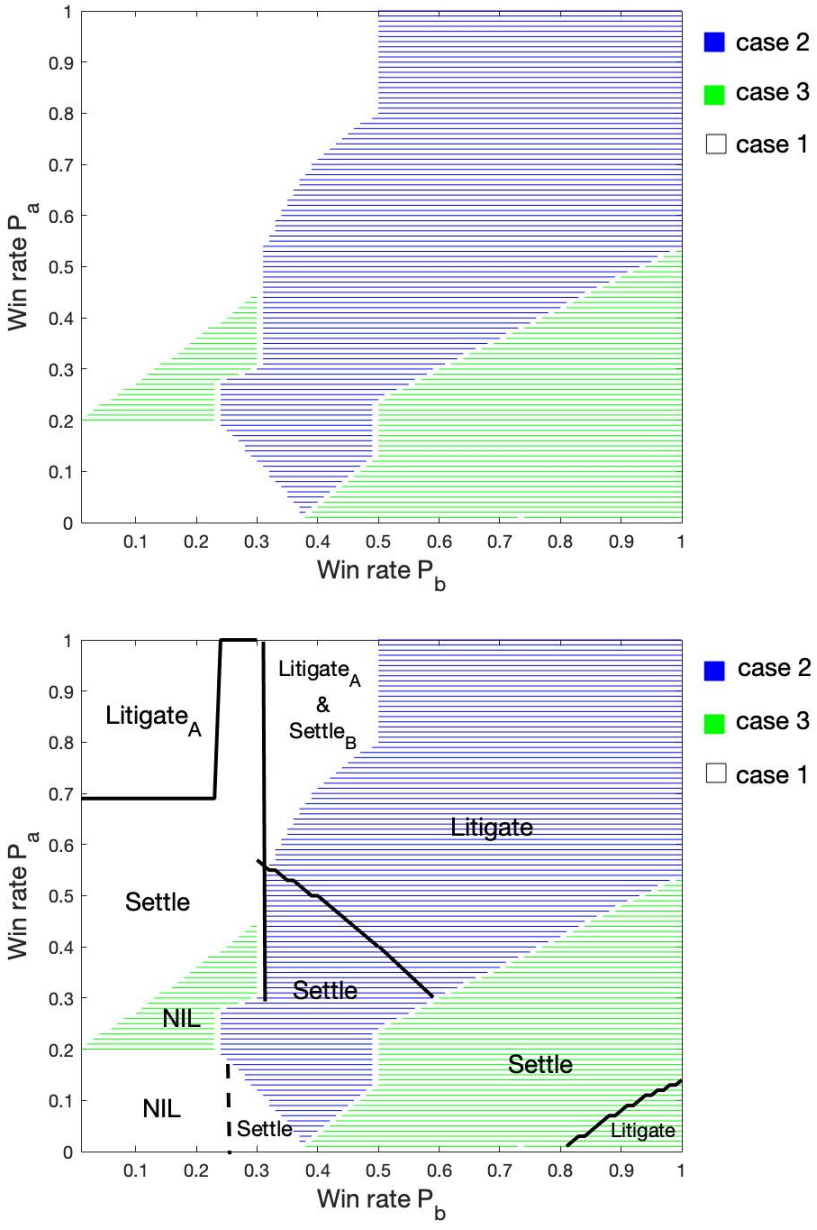


Figure 8: Firms' Incentives to Innovate

The graph shows firms' incentives to innovate if firm B can sleep the patents or no sleeping patents with varying P_b and P_a. The blank area is the region where both firms are willing to innovate. The dotted lines shaded area shows firm B has no incentive to innovate and the solid lines shade region represents firm A has no incentive to innovate.

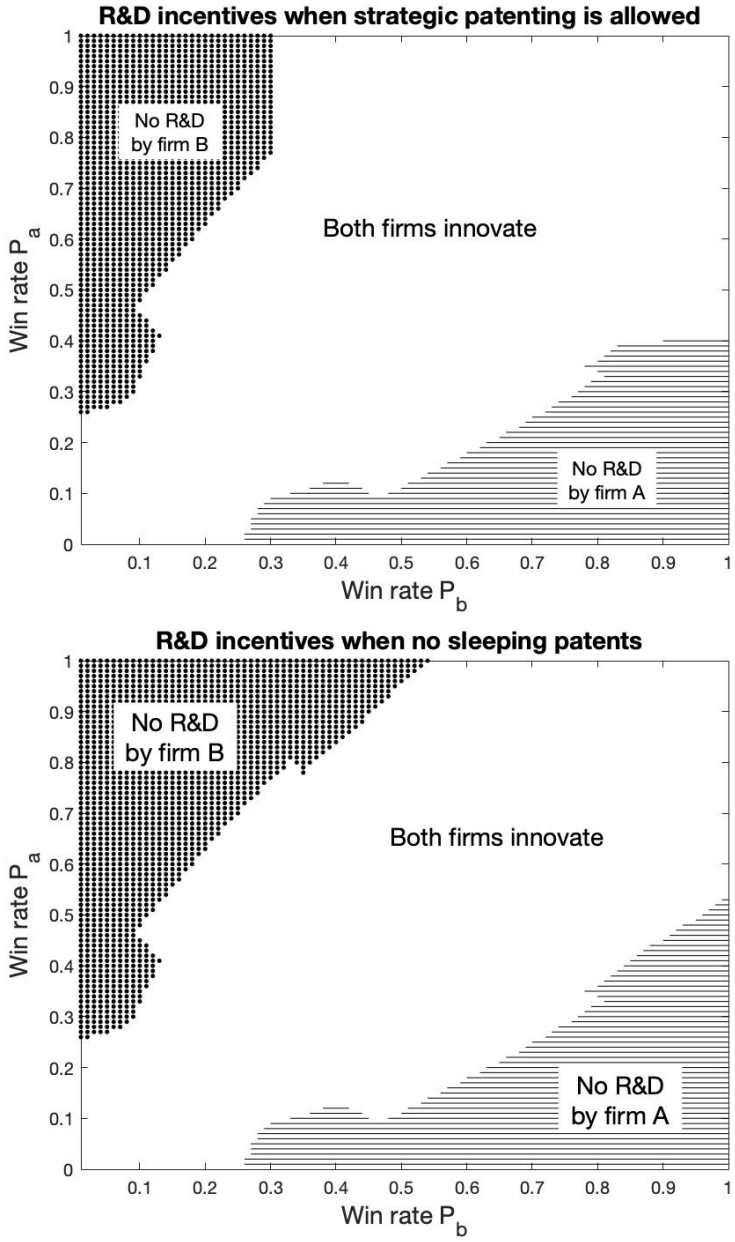


Figure 9: Market Competition and Firms' M&A Decisions

The graphs show the M&A outcomes with different profits if (I) in the base case, (II) the monopoly profit is higher than that in the based case (i.e. $\pi^m = 13 > 10$), (III) the overall duopoly profits are higher than that in the base case (i.e. $\pi^d = 9 > 8$) and (IV) the overall profits if B commercialises are higher than that in the base case (i.e. $\pi^0 = 7.5 > 6$). The blue area is the region where two patent-holding firms A and B merge (i.e. Case 2) and the green area is the region where the sleeping-patent holder B and the non-patent holder C merge (i.e. Case 3). The blank region is the area where there is no M&A (i.e. Case 1).

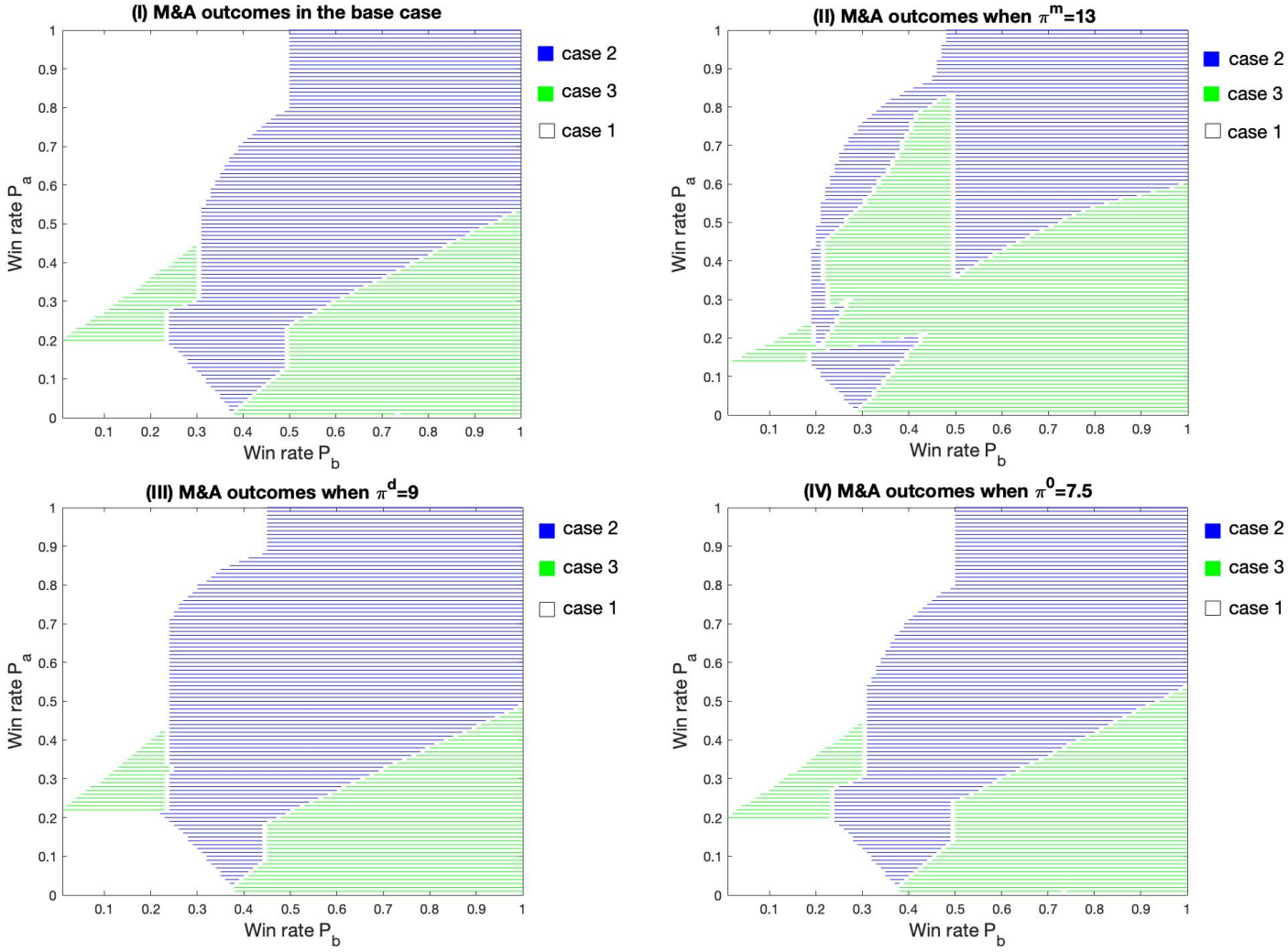
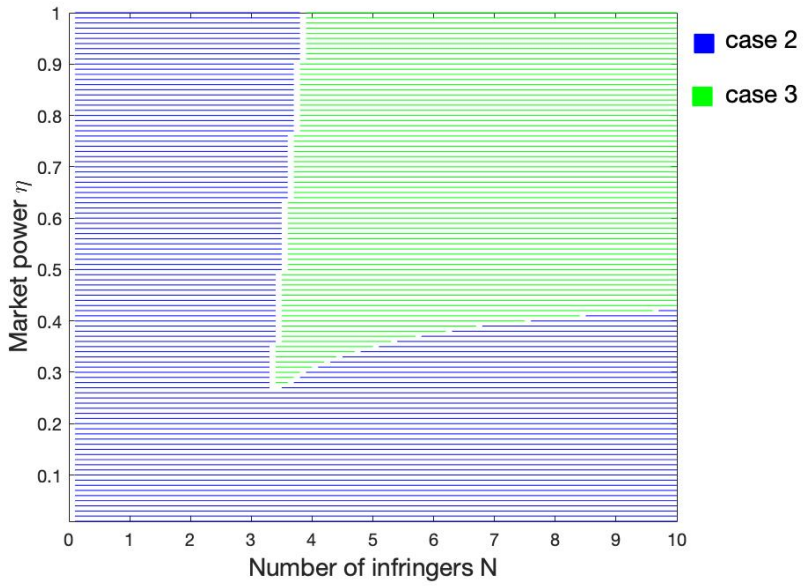


Figure 10: M&A Outcomes with Multiple Infringers

The graph shows firms' M&A decisions with the number of infringers N and the market power of the biggest challenger η. The blue area is the region where two patent-holding firms A and B merge (i.e. Case 2) and the green area is the region where the sleeping-patent holder B and the non-patent holder C merge (i.e. Case 3).



A.1 Proofs

A.1.1 The proof of firms' value function in Case 2

Proof. **Litigation**

Each firm's value if litigation occurs can be written as

$$V_{ab}^{sue} = v_{ab} + P_m \pi^m + (1 - P_m)\pi_a^d - L, \tag{48}$$

$$V_c^{sue} = v_c + P_m \times 0 + (1 - P_m)\pi_c^d - L. \tag{49}$$

The value functions for both parties include three parts. The first term in each of these two expressions are the firm's other revenues that are unrelated to the patents, the second term and the third term are the expected profit in court ruling, and the last term is the litigation cost.

The litigation strategy is optimal for the merged firm if and only if the value of litigating is higher then the value of not litigating. I use V_{ab}^{nl} to denote the value of not litigating for the merged firm and can be expressed as

$$V_{ab}^{nl} = v_{ab} + \pi_a^d. \tag{50}$$

Therefore, the merged firm chooses to litigate if $V_{ab}^{sue} \geq V_{ab}^{nl}$, which gives the litigation condition as follows

$$P_m \geq \frac{L}{\pi^m - \pi_a^d},$$

which indicates that the merged firms' incentives to litigate increase with the market competition $\pi_m - \pi_d$ due to the possibility of regaining the monopoly power and decrease with the litigation cost L.

Ex-ante settlement

When litigation is a credible threat, i.e., $P_m \geq \frac{L}{\pi^m - \pi_a^d}$, the merged firm AB and firm C can settle to resolve the dispute with royalty payment r_2, instead of the litigation. The firms' values for the two parties if settlement occurs can be expressed as

$$V_{ab}^{settle} = v_{ab} + \pi_a^d + r_2 - c, \tag{51}$$

$$V_c^{settle} = v_c + \pi_c^d - r_2 - c. \tag{52}$$

Settlement occurs if the two parties reach an agreement about the royalty payment. On the one hand, the merged firm is willing to settle if the royalty payment r_2 is higher than the minimum royalty payment that he is willing to accept, which is denoted by r_2^{min}. This minimum royalty payment is the royalty payment that makes the value of settlement the same as the value of litigating for the merged firm, i.e., $v_{ab} + \pi_a^d + r_2^{min} - c = v_{ab}^{sue}$. On the other hand, the challenger C is willing to settle if the royalty payment r_2 is lower than the maximum royalty payment he is willing to pay, which is denoted by r_2^{max}. This is the royalty payment that makes the value of settlement is the same as the value of litigating for the challenger, i.e., $v_c + \pi_c^d - r_2^{max} - c = v_c^{sue}$. The minimum

royalty payment and the maximum royalty payment can be expressed as

$$r_2^{min} = P_m(\pi^m - \pi_a^d) - (L - c) \tag{53}$$

$$r_2^{max} = P_m\pi_c^d + (L - c) \tag{54}$$

For the minimum royalty payment, the first term is the expected value the plaintiff can get through patent litigation and the second term is the cost saving from the settlement. For the maximum royalty payment, the first term is also the expected value if patent litigation occurs and the second term is the cost saving in litigation. By settling, both firms save the litigation costs but the plaintiff loses the possibility of gaining monopoly profit, while the defendant can continue to share the market profit.

I use Nash bargaining to solve the royalty payment, taking into account the values if litigation occurs, i.e., $\max_{r_2} = [V_{ab}^{settle} - V_{ab}^{sue}]^{\frac{1}{2}}[V_c^{settle} - V_c^{sue}]^{\frac{1}{2}}$. Substituting the firm values, I have

$$\max_{r_2} = [r_2 - c - (P_m(\pi^m - \pi_a^d) - L)]^{\frac{1}{2}}[\pi_c^d - r_2 - c - ((1 - P_m)\pi_c^d - L)]^{\frac{1}{2}}.$$

Thus, the royalty payment can be expressed as

$$r_2 = \frac{1}{2}(r_2^{min} + r_2^{max}). \tag{55}$$

The optimal royalty payment r_2 is the weighted average of the maximum royalty payment the defendant can pay and the minimum royalty payment the plaintiff can accept to not go through the patent litigation. Since I assume the equal bargaining power for firms, the weight is $\frac{1}{2}$.

Rewrite Eq(55), I have

$$r_2 = \overbrace{P_m\pi_c^d}^{\text{C' expected value in litigation}} + \frac{1}{2}\underbrace{P_m(\pi^m - \pi^d)}_{\text{total foregone revenue in judgment}}.$$

The above expression demonstrates that the royalty payment in settlement is divided into two parts. The first is the expected value if litigation starts for firm C and the second is the half of total expected change in profits earned from the product as a result of the litigation judgement. Therefore, the royalty payment in settlement obtained by the plaintiff is the expected value in litigation for firm C and the divided joint revenues in judgement based on the bargaining power, which is $\frac{1}{2}$.

It is always optimal to settle with the optimal royalty payment r_2 when the litigation is a credible threat for both parties and $r_2^{min} \leq r_2^{max}$, i.e,

$$\overbrace{2(L - c)}^{\text{total cost saving from settlement}} \geq \underbrace{P_m(\pi^m - \pi^d)}_{\text{total foregone revenue in judgment}}. \tag{56}$$

The above inequality (56) shows that settlement is more likely when the joint cost saving from settlement for two firms is large enough and the difference between monopoly profit and the sum of duopoly profits is low. Therefore, the settlement condition expressed via P_m is

$$P_m \in [\frac{L}{\pi^m - \pi_a^d}, \frac{2(L - c)}{\pi^m - \pi^d}].$$

□

A.1.2 The proof of firms' value functions in Case 3

Proof. **Litigation**

If no litigation occurs, firms' values of not litigating are

$$V_a^{nl} = v_a + \pi_a^d, \tag{57}$$
$$V_{bc}^{nl} = v_{bc} + \pi_c^d. \tag{58}$$

The potential outcomes of the litigation, in this case, are as follows. (1) With probability $(1-P_a)(1-P_b)$, neither the merged firm BC nor firm A is ruled as an infringer. This leads to duopoly profits π_a^d for firm A and π_c^d for the merged firm BC. (2) One of the firms is found to have infringed on the other firm's patents. The infringer is forced out of the market whilst the winning party becomes a monopolist. In particular, with probability $P_b(1-P_a)$, the merged firm BC wins the case and gains the monopoly profit while the patent holder A leaves the market. With probability $P_a(1-P_b)$, the patent holder A wins and gets the monopolistic power but the merged firm BC is excluded from the market. (3) With probability P_aP_b, both firms are ruled to have infringed on the other's patents. In this case, two firms form the ex-post cross-licensing agreement by paying the cost of cross-licensing only and continue to share the market. By considering all possibilities, firms' values of litigation can be expressed as

$$V_a^{sue} = v_a + (1-P_a)(1-P_b)\pi_a^d + P_aP_b(\pi_a^d - k) + P_a(1-P_b)\pi^m - L, \tag{59}$$
$$V_{bc}^{sue} = v_{bc} + (1-P_a)(1-P_b)\pi_c^d + P_aP_b(\pi_c^d - k) + P_b(1-P_a)\pi^m - L. \tag{60}$$

On the one hand, firm A litigates when the value of litigating is higher than the value of not litigating, taking into account the possibility of being counter-sued, that is $V_a^{sue} \geq V_a^{nl}$:

$$v_a + (1-P_a)(1-P_b)\pi_a^d + P_aP_b(\pi_a^d - k) + P_a(1-P_b)\pi^m - L \geq v_a + \pi_a^d. \tag{61}$$

Solving (61), I obtain the litigation condition expressed via (P_a, P_b) for firm A, that is

$$\mathcal{L}_a = \{(P_a, P_b) | P_a[(1-P_b)\pi^m + (2P_b - 1)\pi_a^d - P_bk] - P_b\pi_a^d - L \geq 0\}.$$

On the other hand, the merged firm BC litigates taking the possibility of being counter-sued into account and proceeds only if the litigation is worthwhile, that is $V_{bc}^{sue} \geq V_{bc}^{nl}$:

$$v_{bc} + (1-P_a)(1-P_b)\pi_c^d + P_aP_b(\pi_c^d - k) + P_b(1-P_a)\pi^m - L \geq v_{bc} + \pi_c^d. \tag{62}$$

Solving (62), I have the litigation condition expressed via (P_a, P_b) for firm B, that is

$$\mathcal{L}_b = \{(P_a, P_b) | P_b[(1-P_a)\pi^m + (2P_a - 1)\pi_c^d - P_ak] - P_a\pi_c^d - L \geq 0\}.$$

Both firms know a counter-litigation is followed if they initiate the patent lawsuit, therefore, a counter-litigation threat by at least one firm is credible if $(P_a, P_b) \in \mathcal{L} = \mathcal{L}_a \cup \mathcal{L}_b$. Otherwise, there will be no litigation because it is

not worthwhile for either firms to start litigation[26].

Ex-ante cross-licensing

When litigation is a credible threat, i.e. $(P_a, P_b) \in \mathcal{L}$, merged firm BC and A can settle by forming ex-ante cross-licensing agreement to avoid litigation costs. In this case, firms settle by entering into a cross-licensing agreement which allows each party to use the technologies covered by its rival's patents[27]. Therefore, the formation of a cross-licensing is allowed without any restrictions and the litigation outcomes affect how the royalties are split between two parties. I assume the royalty payment is paid by the other patent holder A to the merged firm and is denoted by r_3. This payment can be negative and the negative payment r_3 indicates that the merged firm pays the licensing fee to the other patent holder. Therefore, the firms' values when ex-ante cross-licensing agreement is formed can be written as:

$$V_{bc}^{settle} = v_{bc} + \pi_c^d + r_3 - k, \tag{63}$$

$$V_a^{settle} = v_a + \pi_a^d - r_3 - k. \tag{64}$$

The settlement royalty can be solved through Nash bargaining by solving $\max[V_{bc}^{settle} - V_{bc}^{sue}]^{\frac{1}{2}}[V_a^{settle} - V_a^{sue}]^{\frac{1}{2}}$, i.e., $\max[v_{bc} + \pi_c^d + r_3 - k - V_{bc}^{sue}]^{\frac{1}{2}}[v_a + \pi_a^d - r_3 - k - V_a^{sue}]^{\frac{1}{2}}$. I obtain the transfer in cross-licensing for the merged firm as follows:

$$r_3 = \frac{1}{2}P_b(1-P_a)(\pi^m - \pi_c^d + \pi_a^d) - \frac{1}{2}P_a(1-P_b)(\pi^m - \pi_a^d + \pi_c^d). \tag{65}$$

The first term of the above expression is similar with the optimal royalty payment in Case 2 as in Eq(55), but the probabilities of excluding the competitor are different. In Case 3, the possibility of excluding the rival for the merged firm occurs only if he wins the case and the other patent holder loses, with probability $P_b(1-P_a)$. Therefore, the first term can be regarded as the weighted average of the maximum royalty and the minimum royalty as well. However, since the counter-sue always occurs, the possibility that the other patent holder can exclude the merged firm should also be considered. The second term, thus, shows the weighted average of the maximum and minimum royalty when the other patent holder A wins. Overall, the royalty payment for the merged firm is the difference between profits if only the merged firm BC wins and the profits if only the firm A wins the case.

Rewrite Eq(65), I have

$$r_3 = \overbrace{[P_a(1-P_b) + P_b(1-P_a)]\pi_a^d - P_a(1-P_b)\pi^m}^{\text{A's expected change in litigation}} + \frac{1}{2}\underbrace{[P_a(1-P_b) + P_b(1-P_a)](\pi^m - \pi^d)}_{\text{total foregone revenue in judgment}}.$$

The expression shows that the optimal royalty payment is made up two parts: the expected change in litigation for the other party and the divided total revenues in court ruling. However, due to the possibility of counter-claim, the overall probability that one of the patent holder can gain monopoly profit is $P_a(1-P_b) + P_b(1-P_a)$. For firm A, with probability $P_b(1-P_a)$, she gains the monopoly profits.

For a credible litigation threat, firms prefer to cross-license if the firms' joint cross-licensing profits are higher

[26] I assume both firms' values of litigating are positive, i.e., $V_{bc}^{sue}(P_a, P_b) \geq 0$ and $V_a^{sue}(P_a, P_b) \geq 0$.
[27] I assume away any additional surplus in cross-licensing.

than the joint expected profits from litigation, that is, if the following condition holds:

$$\overbrace{2(L+P_aP_bk-k)}^{\text{total cost saving from settlement}} \geq \underbrace{[P_a(1-P_b)+P_b(1-P_a)](\pi^m-\pi^d)}_{\text{total foregone revenue in judgment}}. \tag{66}$$

Let S be the set of (P_a, P_b) for which the above condition (66) holds. Litigation takes place if and only if $(P_a, P_b) \in \mathcal{L} \setminus S$. The left-hand side of inequality (66) shows the joint cost-saving from ex-ante cross-licensing and the right-hand side shows expected revenues of going through patent litigation. In general, ex-ante cross-licensing is worthwhile if the total cost saving is higher than the additional profit earned from patent litigation. Ex-ante cross-licensing negotiation is more likely when the monopoly profit is not sufficiently greater than the sum of duopoly profits i.e. when product market competition is weak.

Combining the two litigation conditions for two patent holders, i.e. inequalities (61) and (62), it shows the condition if both firms have the incentive to litigate (i.e. $(P_a, P_b) \in (\mathcal{L}_a \cap \mathcal{L}_b)$), then we have

$$[P_a(1-P_b)+P_b(1-P_a)](\pi^m-\pi^d) - 2P_bP_ak \geq 2L. \tag{67}$$

The left-hand side of inequality(67) is the total revenues that firms can obtain through litigation with counter-sue and the right-hand side is the joint cost of going through litigation. By comparing the two inequalities (67) and (66), it can be easily verified that when both firms have the incentive to litigate, ex-ante cross-licensing is impossible.

Moreover, comparing the settlement condition in Case 3, i.e. inequality (66) with the settlement condition in Case 2, i.e. inequality (56), the two expressions show the settlement is more likely in Case 3 because the expected profit from court ruling is lower in Case 3. □

A.1.3 The proof of sequential litigation when no M&A

Proof. In this case, firm A can file the lawsuit against firm C with win rate P_a. Since firm B is not producing, firm B can sue firm A and firm C with a common win rate of P_b because firm A and C are producing homogeneous product, without worrying about the possibility of being counter-sued by firm A. In 2011, Section 299 of the AIA modified the standard for multiple defendants in one patent infringement suits, restricting the multi-defendant suits filed by non-practising entities. Therefore, firm B sues firm A and C separately. According to Choi and Gerlach (2017), if firm B wins, he obtains the maximum damage fees by solving the maximisation problem in Nash bargaining. Since firms have equal bargaining power (i.e.$\frac{1}{2}$) as assumed, the damage fee is determined by solving the maximisation problem. For simplicity, I take the damage fees obtained by firm B to be this maximum value. Specifically, if firm A and C are both in the market with profit π_a^d and π_c^d when firm B wins the litigation, then the maximisation problem is $\max_{d_i}[d_i]^{\frac{1}{2}}[\pi_i^d - d_i]^{\frac{1}{2}}$, which gives $d_i = \frac{\pi_i^d}{2}$. Otherwise, if firm A successfully excludes firm C before firm B litigates, then firm A and B negotiate the damage fees by solving $\max_{d_a}[d_a]^{\frac{1}{2}}[\pi^m - d_a]^{\frac{1}{2}}$, i.e. $d_a = \frac{\pi^m}{2}$ and firm C pays no damage fees. Therefore, the damage fees claimed from firm i can be expressed as follows:

$$d_a = \begin{cases} \frac{\pi^m}{2}, & \text{if A excludes C} \\ \frac{\pi_a^d}{2}. & \text{Otherwise} \end{cases} \tag{68}$$

$$d_c = \begin{cases} 0, & \text{if A excludes C} \\ \frac{\pi_c^d}{2}. & \text{Otherwise} \end{cases} \tag{69}$$

If firm A wins the litigation with probability P_a, firm C is out of the market and thus firm B cannot gain any licensing fees in this case. If firm A loses with probability $1 - P_a$, however, firm C earns the duopoly profit as before and need to pay the damage fees to firm B in the next stage.

By considering the probability of different patent litigation outcomes in if firm A litigates first, the joint expected damage fees earned by firm B if firm A starts litigation first is

$$\overbrace{P_a \frac{\pi^m}{2}}^{\text{expected damage fees if A wins}} + \underbrace{(1-P_a)\frac{\pi_a^d}{2}}_{\text{expected damage fees if A loses}}$$

The joint expected damage fee received by firm B if no litigation is initiated by firm A or firm B litigates before A's litigation is $\frac{\pi^d}{2}$. Thus, firm B's joint expected damage fees if A starts litigation first is higher than his joint expected damage fees if litigation has not yet been initiated by A, i.e., $\frac{(1-P_a)\pi^d + P_a\pi^m}{2} \geq \frac{\pi^d}{2}$. This shows that it is optimal for firm B to wait for the patent litigation outcome between A and C before starting their lawsuits.

Moreover, the value of firms if B litigates firm i only can be written as follows:

$$V_b^{sue} = v_b + P_b d_i - L, \tag{70}$$

$$V_i^{sue} = v_i + (1-P_b)\pi_i^d + P_b(\pi_i^d - d_i) - L = v_i + \pi_i^d - P_b d_i - L, \tag{71}$$

Note if both firm A and C are being sued, the firm B pays litigation cost $L^0 \in (L, 2L)$ because the plaintiff can save some costs as the two cases are similar, and get the expected damage fees $P_b(d_a + d_c)$.

The first term v_i is the value of other revenues and $P_b d_i$ is the expected damage fee if B litigates. Firm B has the incentive to litigate if the value of litigating is higher than the value of not litigating, that is, $V_b^{sue} > v_b$, which gives the litigation condition, i.e., $P_b \geq \frac{L}{d_i}$. For a given credible litigation threat, both the plaintiff (firm B) and the defendant (firm A or C) would agree to settle out of court by paying the licensing fee $P_b d_i$ with settlement cost $c < L$ and $2c < L^0$ and d_i follows the expression in Eq(68) and Eq(69). In the case when the patent holding firm A excludes the challenger C in the first stage, firm B does not settle with firm C. Since firm B cannot exclude other infringers, it is optimal to settle to save costs instead of going through patent litigation. The settlement feasibility condition, thus, is the same with the litigation condition, i.e., $\geq \frac{L}{d_i}$. Rewriting this settlement feasibility condition, I have $P_b d_i \geq L$. Therefore, settlement occurs when the royalty payment is higher than the litigation cost.

Therefore, the two stage sequential game is modelled as follows: in the first stage, the patent-holding firm A chooses and the non-patent holder C decide their optimal strategy based on firm A's win rate P_a, i.e. do nothing, litigate or settle. In the next stage, the non-producing firm B choose whether to settle with the firm still in the market. Specifically. if firm A continues litigate, firm B will choose to settle with credible litigation threat after knowing the court ruling result between firm A and C. □

A.1.4 The proof of firms' values in Case 1 depending on the royalty as in Eq(5)

Proof. **Case 1.1: Firm B is unable to settle**

Firstly, if $P_b < \frac{2L}{\pi^m}$, firm B is unable to obtain the royalty payment in the next stage from both firms in any case. Firms' values of litigation in the first stage are

$$V_a^{sue} = v_a + (1 - P_a)\pi_a^d + P_a\pi^m - L, \tag{72}$$

$$V_c^{sue} = v_c + (1 - P_a)\pi_c^d - L, \tag{73}$$

$$V_b^{sue} = v_b. \tag{74}$$

and the values of not litigating are

$$V_a^{nl} = v_a + \pi_a^d \tag{75}$$

$$V_c^{nl} = v_c + \pi_c^d, \tag{76}$$

$$V_b^{nl} = v_b. \tag{77}$$

Firm A starts litigation if the value of litigating is higher than the value of not litigating, i.e., $V_a^{sue} \geq V_a^{nl}$, which gives the litigation condition

$$P_a \geq \frac{L}{\pi^m - \pi_a^d}.$$

For a given credible litigation threat, firm A and C can settle with r_{none}^1 to save cost by solving the maximisation problem

$$\max_{r_{none}^1} [\pi_a^d + r_{none}^1 - c - [(\pi_a^d + P_a(\pi^m - \pi_a^d)) - L])]^{\frac{1}{2}} [\pi_c^d - r_{none}^1 - c - ((1 - P_a)\pi_c^d - L)]^{\frac{1}{2}},$$

which gives the expression of royalty payment

$$r_{none}^1 = \overbrace{P_a\pi_c^d}^{\text{C's expected value in litigation}} + \frac{1}{2} \underbrace{P_a(\pi^m - \pi^d).}_{\text{total foregone revenue in judgment}}$$

In this case, if A starts patent litigation, she has the possibility P_a to gain the full monopoly profit without worrying the litigation risk from firm B, thus the joint profits in litigation for both parties are $P_a(\pi^m - \pi^d)$.

Settlement occurs if the value of settlement is higher than the value of litigating, that is,

$$\overbrace{2(L - c)}^{\text{total cost saving from settlement}} \geq \underbrace{P_a(\pi^m - \pi^d).}_{\text{total foregone revenue in judgment}}$$

Therefore, firms settle if $P_a \leq \frac{2(L-c)}{(\pi^m - \pi^d)}$.

Case 1.2: Firm B settles with the monopolist A

Secondly, firm B has the incentive to settle with the monopolist A only when firm A excludes C in the first stage, i.e., $P_b \in [\frac{2L}{\pi^m}, \frac{2L}{\pi_a^d})$. When patent-holding firm A wins the litigation and gains monopoly profit π^m in the first stage, the expected royalty payment that firm B can obtain is $r_b^a = \frac{P_b}{2} P_a \pi^m$ with expected cost $P_a c$.

Firms' values of litigation in the first stage can be written as

$$\hat{V}_a^{sue} = v_a + (1-P_a)\pi_a^d + (1-\frac{P_b}{2})P_a\pi^m - P_a c - L, \tag{78}$$

$$V_c^{sue} = v_c + (1-P_a)\pi_c^d - L, \tag{79}$$

$$\hat{V}_b^{sue} = v_b + \frac{P_b}{2}P_a\pi^m - P_a c, \tag{80}$$

and the values of not litigating are

$$V_a^{nl} = v_a + \pi_a^d \tag{81}$$

$$V_c^{nl} = v_c + \pi_c^d, \tag{82}$$

$$V_b^{nl} = v_b. \tag{83}$$

Firm A litigates if the value of litigating is higher than the value of not litigating, i.e., $(1-P_a)\pi_a^d + (1-\frac{P_b}{2})P_a\pi^m - P_a c - L \geq \pi_a^d$. The litigation condition thus is $P_a \geq \frac{L}{(1-\frac{P_b}{2})\pi^m - \pi_a^d - c}$.

For a given credible litigation threat, firm A and C can settle to resolve the dispute with royalty payment r_1^{am}. Here the notation am means firm B settles with A only if firm A is the monopolist. The value of settlement can be expressed as

$$V_a^{settle} = v_a + \pi_a^d + r_1^{am} - c \tag{84}$$

$$V_c^{settle} = v_c + \pi_c^d - r_1^{am} - c, \tag{85}$$

$$V_b^{settle} = v_b. \tag{86}$$

Solving the maximum problem

$$\max_{r_1^a}[V_a^{settle} - \hat{V}_a^{sue}]^{\frac{1}{2}}[V_c^{settle} - V_c^{sue}]^{\frac{1}{2}},$$

i.e.,

$$\max_{r_1^{am}}[\pi_a^d + r_1^{am} - c - [(1-P_a)\pi_a^d + (1-\frac{P_b}{2})P_a\pi^m - P_a c - L])]^{\frac{1}{2}}[\pi_c^d - r_1^{am} - c - ((1-P_a)\pi_c^d - L)]]^{\frac{1}{2}},$$

I obtain the royalty payment in settlement

$$r_1^{am} = \overbrace{P_a\pi_c^d}^{\text{C's expected value in litigation}} + \frac{1}{2}\underbrace{P_a((1-\frac{P_b}{2})\pi^m - \pi^d - c).}_{\text{total foregone revenue in judgment}}$$

Settlement occurs if the value of settlement is higher than the value of litigating, that is,

$$\overbrace{2(L-c)}^{\text{total cost saving from settlement}} \geq \underbrace{P_a((1-\frac{P_b}{2})\pi^m - \pi^d - c).}_{\text{total foregone revenue in judgment}}$$

If firm A and C settle and share market profits in the first stage, both firms save the litigation cost, thus the joint cost saving from settlement is $2(L-c)$. The expected revenue in judgement by considering the licensing

fees paid in the next stage is the difference of expected value for two firms if litigation occurs and the value if no litigation occurs, that is,

$$[P_a(\pi^m - d_a - c) + (1 - P_a)\pi^d] - \pi^d = P_a((1 - \frac{P_b}{2})\pi^m - \pi^d - c).$$

Settlement is worthwhile for both firm A and C when the values of settlement are higher than the values of litigation, which gives the settlement condition $P_a \leq \frac{2(L-c)}{(1-\frac{P_b}{2})\pi^m - \pi^d - c}$.

Case 1.3: Firm B settles with firm A

Thirdly, I consider the case if firm B only has the incentive to settle with firm A if $P_b \in [\frac{2L}{\pi_a^d}, \frac{2L^0}{\pi^d})$. In this case, firm B's expected royalty payment from firm A is $\frac{P_b}{2}[(1 - P_a)\pi_a^d + P_a\pi^m]$.

Firms' values of litigation between firm A and C in the first stage are

$$\hat{V}_a^{sue} = v_a + (1 - \frac{P_b}{2})((1 - P_a)\pi_a^d + P_a\pi^m) - c - L, \tag{87}$$

$$V_c^{sue} = v_c + (1 - P_a)\pi_c^d - L, \tag{88}$$

$$\hat{V}_b^{sue} = v_b + \frac{P_b}{2}((1 - P_a)\pi_a^d + P_a\pi^m) - c. \tag{89}$$

and the values of not litigating can be written as

$$\hat{V}_a^{nl} = v_a + (1 - \frac{P_b}{2})\pi_a^d - c \tag{90}$$

$$V_c^{nl} = v_c + \pi_c^d, \tag{91}$$

$$\hat{V}_b^{nl} = v_b + \frac{P_b}{2}\pi_a^d - c. \tag{92}$$

Firm A litigates if the value of litigating is higher than his value of not litigating, which is the same with the previous case, i.e., $P_a \geq \frac{L}{(1-\frac{P_b}{2})(\pi^m - \pi_a^d)}$.

When litigation occurs, firm A and C can settle with r_a^1 in Nash bargaining and the values of settlement are

$$\hat{V}_a^{settle} = v_a + (1 - \frac{P_b}{2})\pi_a^d + r_a^1 - 2c \tag{93}$$

$$V_c^{settle} = v_c + \pi_c^d - r_a^1 - c, \tag{94}$$

$$\hat{V}_b^{settle} = v_b + \frac{P_b}{2}\pi_a^d - c. \tag{95}$$

Solving the maximisation problem

$$\max_{r_1^a}[\hat{V}_a^{settle} - \hat{V}_a^{sue}]^{\frac{1}{2}}[V_c^{settle} - V_c^{sue}]^{\frac{1}{2}},$$

i.e.,

$$\max_{r_1^a}\big[(1 - \frac{P_b}{2})\pi_a^d + r_1^a - 2c - [(1 - \frac{P_b}{2})(\pi_a^d + P_a(\pi^m - \pi_a^d)) - c - L]\big]^{\frac{1}{2}}$$
$$\times \big[\pi_c^d - r_1^a - c - ((1 - P_a)\pi_c^d - L)\big]^{\frac{1}{2}},$$

I obtain the royalty payment

$$r_1^a = \overbrace{P_a\pi_c^d}^{\text{C's expected value in litigation}} + \frac{1}{2}\underbrace{P_a[(1-\frac{P_b}{2})(\pi^m-\pi_a^d)-\pi_c^d]}_{\text{total foregone revenue in judgment}}.$$

Settlement occurs if the value of settling is higher than the value of litigating, i.e.,

$$\overbrace{2(L-c)}^{\text{total cost saving from settlement}} \geq \underbrace{P_a[(1-\frac{P_b}{2})(\pi^m-\pi_a^d)-\pi_c^d]}_{\text{total foregone revenue in judgment}}.$$

The joint cost saving from settlement in this case is the same with Case 1.1. The joint revenues in judgement in this case is

$$\overbrace{P_a(\pi^m-d_a-c)}^{\text{value if A wins}} + \underbrace{(1-P_a)(\pi_a^d-d_a-c+\pi_c^d)}_{\text{value if A loses}} - \overbrace{[(\pi_a^d-d_a-c)+\pi_c^d]}^{\text{value if no litigation}}$$
$$= P_a[(1-\frac{P_b}{2})\pi^m-\pi^d].$$

Settlement is worthwhile for both firm A and C when the values of settlement are higher than the values of litigation, which gives the settlement condition

$$P_a \leq \frac{2(L-c)}{(1-\frac{P_b}{2})(\pi^m-\pi_a^d)-\pi_c^d}.$$

Case 1.4: Firm B settles with both firms

Finally, if $P_b \geq \frac{2L^0}{\pi^d}$, firm B is able to obtain the royalty payment from both firms. If the patent holder A wins the litigation and gains the monopoly profit π^m in the first stage, firm A pays the licensing fees $\frac{P_b}{2}\pi^m$ to the other patent holder B in the next stage. If the patent holder A loses the case and shares the market profit with the challenger C in the first stage, both firms pay the licensing fees to firm B, that is $\frac{P_b}{2}\pi_a^d$ and $\frac{P_b}{2}\pi_c^d$ for firm A and firm C respectively[28].

Firms' value functions of litigation taking into account the licensing fees paid in the next stage can be expressed as follows:

$$\hat{V}_a^{sue} = v_a + (1-\frac{P_b}{2})((1-P_a)\pi_a^d + P_a\pi^m) - c - L \tag{96}$$
$$\hat{V}_c^{sue} = v_c + (1-\frac{P_b}{2})(1-P_a)\pi_c^d - (1-P_a)c - L, \tag{97}$$
$$\hat{V}_b^{sue} = v_b + \frac{P_b}{2}((1-P_a)\pi^d + P_a\pi^m) - (2-P_a)c. \tag{98}$$

The above expressions for firm A and firm C constitute four components. The first term is the value of other revenues and this value is large enough to pay all costs needed. The second term is the expected revenue of litigation in the first stage by considering the licensing fees paid in the next stage, which is r_b^i. The third term is

[28]For simplicity, I assume when firm B settle with firm A or firm C, he considers the infringing profits based on the litigated patents are fixed, which can either be duopoly profits π_i^d or the monopoly profit π^m without considering the costs paid in the first stage.

the cost of settlement in the next stage and the last term is the litigation cost in the first stage. Firm B's value function also has the first three components, but there is no fourth term in the expression because he is not involved in patent litigation.

If firm A does not litigate, firm B is still able to settle with firms with the credible litigation threat by paying cost c, thus firms' values are

$$\hat{V}_a^{nl} = v_a + (1 - \frac{P_b}{2})\pi_a^d - c \tag{99}$$

$$\hat{V}_c^{nl} = v_c + (1 - \frac{P_b}{2})\pi_c^d - c, \tag{100}$$

$$\hat{V}_b^{nl} = v_b + \frac{P_b}{2}\pi^d - 2c. \tag{101}$$

Therefore, firm A starts litigation if the value of litigating is higher than the value of not litigating by considering the licensing fees paid in the next stage, i.e.,

$$\hat{V}_a^{sue} - \hat{V}_a^{nl} = (1 - \frac{P_b}{2})((1 - P_a)\pi_a^d + P_a\pi^m) - (1 - \frac{P_b}{2})\pi_a^d = (1 - \frac{P_b}{2})P_a(\pi^m - \pi_a^d) - L \geq 0,$$

which gives the litigation condition $P_a \geq \frac{L}{(1-\frac{P_b}{2})(\pi^m - \pi_a^d)}$.

For a given credible litigation threat, firm A and C can settle before litigation to save cost and the royalty payment is denoted by r_1^{both}. Once firms settle, firm B can still gain the royalty fee $\frac{\pi^d}{2}$ as the total duopoly profit remains the same. Firms' values if settlement occurs, thus, can be expressed as

$$\hat{V}_a^{settle} = v_a + (1 - \frac{P_b}{2})\pi_a^d + r_1^{both} - 2c \tag{102}$$

$$\hat{V}_c^{settle} = v_c + (1 - \frac{P_b}{2})\pi_c^d - r_1^{both} - 2c, \tag{103}$$

$$\hat{V}_b^{settle} = v_b + \frac{P_b}{2}\pi^d - 2c. \tag{104}$$

I obtain r_1^{both} by solving the maximisation problem

$$\max_{r_1^{both}} [\hat{V}_a^{settle} - \hat{V}_a^{sue}]^{\frac{1}{2}} [\hat{V}_c^{settle} - \hat{V}_c^{sue}]^{\frac{1}{2}}.$$

Substituting the firm values in settlement or litigation, I have the royalty payment

$$r_1^{both} = \overbrace{P_a(1 - \frac{P_b}{2})\pi_c^d}^{\text{C's expected value in litigation}} + \frac{1}{2}\underbrace{P_a[(1 - \frac{P_b}{2})(\pi^m - \pi^d) - c]}_{\text{total foregone revenue in judgment}}.$$

Settlement is worthwhile if the value of litigating is higher than the value of settlement, that is,

$$\overbrace{2(L - c)}^{\text{total cost saving from settlement}} \geq \underbrace{P_a(1 - \frac{P_b}{2})(\pi^m - \pi^d) + P_a c}_{\text{total foregone revenue in judgment}}.$$

The above condition shows that settlement is worthwhile if the joint cost savings from settlement for both firm A and firm B are higher than the total forgone revenue in judgement. This total forgone revenue in judgement is the

expected change in profits earned from the product as a result of the litigation judgement, and can be expressed as follows

$$\overbrace{P_a(\pi^m - d_a - c)}^{\text{value if A wins}} + \underbrace{(1-P_a)(\pi_a^d - d_a - c + \pi_c^d - d_c - c)}_{\text{value if A loses}} - \overbrace{[(\pi_a^d - d_a - c) + (\pi_c^d - d_c - c)]}^{\text{value if no litigation}}$$
$$= P_a[(1 - \frac{P_b}{2})(\pi^m - \pi^d) + c].$$

Therefore, firms settle if $P_a \leq \frac{2(L-c)}{(1-\frac{P_b}{2})(\pi^m - \pi^d) + P_a c}$.

If the settlement cost c is small enough, it is easy to verify that $r^1_{none} \geq r^1_{a*} \geq r^1_a \geq r^1_{both}$. Overall, firm A chooses his optimal patent litigation strategy by considering the litigation threat by firm B.

□

A.1.5 The proof of firms' values in Case 1 with multiple infringers

Proof. Firm A and the biggest challenger C's values depend on their strategies in patent litigation. Two firms' values of not litigating are

$$V_a^{nl} = v_a + \pi_a, \tag{105}$$
$$V_c^{nl} = v_c + \pi_c(\eta + \frac{1-\eta}{N}). \tag{106}$$

Firm A's value of litigating against all infringers[29] can be expressed as

$$V_a^{sue} = v_a + \pi_a + P_a(\pi^m - \pi_a) - NL. \tag{107}$$

and firm C's value if A litigates is

$$V_c^{sue} = v_c + (1-P_a)\pi_c(\eta + \frac{1-\eta}{N}) - L \tag{108}$$

Therefore, litigation is worthwhile for firm A if the value of litigating is higher than the value of not litigating, that is, $P_a \geq \frac{NL}{\pi^m - \pi_a}$.

The patent holder A can choose to settle with all infringers if the following condition holds, that is

$$\overbrace{2N(L-c)}^{\text{cost saving from settlement}} \quad \geq \quad \underbrace{P_m(\pi^m - \pi^{n+1})}_{\text{foregone revenue in judgment}} .$$

The overall royalty payment is $r_1 = P_a\pi_c + \frac{1}{2}(\pi^m - \pi^{n+1})$ and C pays the royalty rate $r_1^c = \pi_c(\eta + \frac{1-\eta}{N})r_1$

The above litigation and settlement conditions show that firm A has less incentive to litigate or settle if the number of infringers N is large.

□

[29] It is possible for the patent holder to start sequential litigation. However, the strategy of sequential litigation also decreases the likelihood of gaining monopoly profit, thus increasing the likelihood of settlement and lowering the royalty payment. Therefore, sequential litigation is not always optimal.

A.1.6 The proof of firms' values in Case 3 with multiple infringers

Proof. If firm B merges with firm C, the merged firm BC chooses whether to cross-license with the other patent holder A with a credible litigation threat by considering going through all lawsuits to exclude other infringers. This case is similar with case 3 discussed in the benchmark model.

The merged firm's value of litigating can be expressed as

$$V_{bc}^{sue} = v_{bc} + (1-P_a)(1-P_b)\pi_c(\eta + \frac{1-\eta}{N}) + P_aP_b(\pi_c^d - k) + P_b(1-P_a)\pi^m - NL,$$

and the merged firm's value if no action is taken can be written as

$$V_{bc}^{nl} = v_{bc} + \pi_c(\eta + \frac{1-\eta}{N}).$$

Litigation is worthwhile for the merged firm BC if the value of litigating is higher than the value of not litigating, i.e. $V_{bc}^{sue} \geq V_{bc}^{nl}$. Solving the inequality gives the litigation condition for the merged firm BC that defined as $\mathcal{L}_c^n$, i.e.,

$$\mathcal{L}_c^n = \{(P_a, P_b) | P_b[(1-P_a)\pi^m + (P_a-1)\pi_c(\eta + \frac{1-\eta}{N}) + P_a(\pi_c^d - k)] - P_a\pi_c(\eta + \frac{1-\eta}{N}) - NL \geq 0\}.$$

Similarly, the other patent holder A's value of litigating can be written as

$$V_a^{sue} = v_a + (1-P_a)(1-P_b)\pi_a + P_aP_b(\pi_a^d - k) + P_a(1-P_b)\pi^m - NL,$$

and firm A's value of not litigating is

$$V_a^{nl} = v_a + \pi_a.$$

The patent-holding firm A starts the litigation if the value of litigating is higher than the value of not litigating, i.e. $V_a^{sue} \geq V_a^{nl}$. This gives the litigation condition for firm A, i.e.,

$$\mathcal{L}_a^n = \{(P_a, P_b) | P_a[(1-P_b)\pi^m + (P_b-1)\pi_a + P_b(\pi_a^d - k)] - P_b\pi_a - NL \geq 0\}.$$

I further investigate how the two parameters: N (the number of infringers) and η (the C's market power) affect both the merged firm BC and firm A's litigation incentives. Let

$$l_c^n(\eta, N) = P_b[(1-P_a)\pi^m + (P_a-1)\pi_c(\eta + \frac{1-\eta}{N}) + P_a(\pi_c^d - k)] - P_a\pi_c(\eta + \frac{1-\eta}{N}) - NL,$$
$$l_a^n(\eta, N) = P_a[(1-P_b)\pi^m + (P_b-1)\pi_a + P_b(\pi_a^d - k)] - P_b\pi_a - NL.$$

I obtain the first derivatives with respect to η and N as follows:

$$\frac{\partial l_c^n(\eta, N)}{\partial \eta} = (1 - \frac{1}{N})[P_bP_a - (P_b + P_a)]\pi_c < 0, \tag{109}$$
$$\frac{\partial l_c^n(\eta, N)}{\partial N} = \frac{1-\eta}{N^2}[(P_b + P_a) - P_bP_a]\pi_c - L. \tag{110}$$

$$\frac{\partial l_a^n(\eta, N)}{\partial \eta} = 0, \tag{111}$$

$$\frac{\partial l_a^n(\eta, N)}{\partial N} = -L < 0. \tag{112}$$

Condition (109) shows the negative impact of market power (i.e. η) on the merged firm's incentives to litigate, and Eq(111) shows no impact of η on the patent holder A's litigation incentives. The asymmetric market power thus reduces the litigation risk, which is consistent with Choi and Gerlach (2017). This is because the high market power gives the biggest challenger enough profit to stay in the market competition instead of litigating to exclude others.

Furthermore, the condition (112) shows a negative impact of the number of infringers on the patent holder A's litigation incentives. However, Eq(110) indicates the impact of the number of infringers on the merged firm's litigation incentive is not monotonic. If the cost of one suit L is small enough, Eq(110) is likely to be positive, leading to a positive impact on the merged firm's litigation incentives. If the cost one suit L is high enough, Eq(110) can be negative and indicates a negative impact on the merged firm's incentive to litigate.

Similar to Case 3 in the benchmark case, litigation occurs if one of the two firms has the litigation incentives. Therefore, the litigation condition is $(P_a, P_b) \in \bar{\mathcal{L}}^n = \bar{\mathcal{L}}_a^n \cup \bar{\mathcal{L}}_b^n$. In this case, firms can form the ex-ante cross-licensing agreement to avoid litigation. I assume the patent holder A pays the licensing fee r^n to the merged firm and each firm pays the cost k. Once the settlement agreement is reached, the two patent-holding firms form joint defence agreement to sue or settle with other infringer simultaneously, which is similar with the case discussed in Section 4.

Therefore, the two patent-holding firms can sue other infringers if the following condition holds

$$\mathcal{L}_m^{cl} = \{(P_a, P_b) | (P_a + P_b - P_a P_b)(\pi^d - \pi_c(\eta + \frac{1-\eta}{N}) - \pi_a) - 2(N-1)L \geq 0\}. \tag{113}$$

They can settle with other infringers $(P_a, P_b) \in \mathcal{L}_m^{cl}$ and the following condition holds

$$S_m^{cl} = \{(P_a, P_b) | 2(L-c) - (P_a P_b - P_b - P_a)(\pi^d - \pi^{n+1}) \geq 0\}. \tag{114}$$

As a result, if settling with other infringers is worthwhile for both patent holders, the feasibility condition for the two firms to form ex-ante cross-licensing agreement in the first stage is

$$\begin{aligned} &\overbrace{2(NL - (N-1)c + P_a P_b k - k)}^{\text{cost saving from settlement}} \geq \\ &\underbrace{[P_a(1-P_b) + P_b(1-P_a)](\pi^m - \pi^d) + \frac{1}{2}(P_a P_b - P_a - P_b)(\pi^d - \pi^{n+1})}_{\text{foregone revenue in judgment}}. \end{aligned} \tag{115}$$

Otherwise, if litigating all infringers is optimal for the two patent holders, the feasibility condition of forming ex-ante cross-licensing between them can be expressed as

$$\overbrace{2(L + P_a P_b k - k)}^{\text{cost saving from settlement}} \geq \underbrace{[P_a(1-P_b) + P_b(1-P_a)](\pi^m - \pi^d)}_{\text{foregone revenue in judgment}}. \tag{116}$$

According to the inequalities (115) and (116), market power η and the number of infringer N do not affect

firms' incentive to settle ex-ante significantly. However, the market competition (i.e. $\pi^m - \pi^d$ and $\pi^m - \pi^{n+1}$) has a negative relationship with the likelihood of ex-ante settlement, thus increasing litigation risks.

□